DemoLab Paper 02/2020

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Abstract

Standard life annuities are attractive mainly for healthy people. Premium rates are consequently kept high. Hence, a large portion of potential annuitants are out of reach of insurers. Lowering the premium rate would of course raise the attractiveness of the life annuity product, but would result in a more heterogeneous portfolio and notably in a likely underpricing in particular for healthy annuitants. Such a solution should then be considered unrealistic. In order to expand their business, some insurers have recently started offering better annuity rates to people whose health conditions are worse than those of likely buyers of standard life annuities. Special-rate life annuity (or underwritten life annuity) products have then been designed and launched. The underwiting step, before policy issue, must include the assessment of the applicant's health conditions and the possible impact of his/her lifestyle, and will then result in assigning the applicant to a specific rating class. Usually, two to four rating classes are defined and implemented. Hence, the special-rate line of business (LoB) is arranged in a set of subportfolios, which, together with the standard life annuity subportfolio, constitutes the life annuity LoB. Premium rates must be determined according to the (assumed) lifetime probability distribution of individuals belonging to the various subportfolios. The worse the health conditions, the smaller the modal age at death (as well as the expected lifetime), but the higher the variance of the lifetime distribution. The latter aspect is due to a significant data scarcity as well as to the mix of possible pathologies leading to each specific rating class. A higher degree of (partially unobservable) heterogeneity follows, inside each subportfolio of special-rate annuities. The variance of each life annuity payout of course impacts on the overall risk profile of the life annuity portfolio. Thus, on the one hand a higher premium income can be expected, on the other a higher variability of the total portfolio payout must be faced, because of both the larger size and the specific high variability of payouts related to special-rate annuities. What about the "balance"? The present research aims at analyzing the impact of extending the life annuity LoB by including special-rate life annuities. Numerical evaluations have been performed by adopting a deterministic approach as well as a

stochastic one, according to diverse assumptions concerning both lifetime distributions and subportfolio sizes. Although results we have obtained clearly depend on assumptions (notably, regarding the lifetime distributions), interesting achievements witness the possibility of extending the business without taking huge amounts of risk. Hence, the Risk Management objective "enhancing the company market share" can be pursued without significant worsening of the LoB risk profile.

Contents

1 Introduction and motivation

Considerable attention is currently being devoted in insurance work (and, in particular, in the actuarial work) to the management of life annuity portfolios and to annuity product design, because of the growing importance of annuity benefits paid by private pension schemes and individual policies.

In particular, the progressive shift in many countries from defined benefit to defined contribution pension schemes has increased the interest in life annuity products with a guaranteed periodic benefit.

Nevertheless, various “weak” features of the (traditional) life annuities should be noted, looking at the product from both the annuity provider’s and the customer’s perspective.

However, many features can be improved by moving from the traditional products to more complex products, for example by adding riders (that is, supplementary benefits), or by adopting restrictions in the age intervals covered, or by allowing for individual risk factors hence “tailoring” the annuity rates (at least to some extent) to specific features of the customer.

Possible classifications according to individual risk factors and consequent calculation of annuity rates are the main topic of this paper. To this purpose, we focus on special-rate life annuity products, on the one hand looking at recent market experience, and, on the other, proposing a specific actuarial setting to assess the risk profile of an annuity portfolio including special-rate life annuities.

From a practical point of view, we are interested in checking whether an enlargement of the annuity portfolio obtained by including special-rate life annuities (and hence a possible increase in the insurer’s market share) does affect the portfolio risk profile.

The remainder of this paper is organized as follows. In Sect. 2 we first describe the basic features of the standard SPIA, that is, the single premium immediate life annuity. We then discuss possible generalizations of the life annuity structure, moving from traditional addition of supplementary benefits, to more recent proposals aiming to make the annuity product more attractive.

Special-rate life annuities are specifically dealt with in Sect. 3. In particular, types of special-rate annuities are described and rating schemes are discussed. Some related market aspects are addressed in Sect. 4.

Sections 5 to 9 constitute the technical core of this research. In particular, the biometric assumptions and the proposed actuarial model are described in Sect. 5 and 6 respectively. Numerical results concerning the portfolio risk profiles are presented and commented in Sects. 7 and 8. In Sect. 9 we quantify assets requirements to face insurer's liabilities.

Some final remarks in Sect. 10 conclude the paper.

2 Innovation in life annuity products

In this section, we first describe the basic features of the standard immediate life annuity. We then discuss possible generalizations of the life annuity structure, ranging from traditional additions of supplementary benefits, to more recent proposals aiming to make the annuity product more attractive.

2.1 SPIA: the basic standard life annuity

The acronym SPIA denotes the *Single Premium Immediate Annuity*, which provides the annuitant with a lifelong sequence of periodic (e.g. annual, or monthly, or semestral) benefits. In particular, referring to the product usually labeled as *standard*, we assume that periodic benefits follow a flat time profile, and that no other benefit is provided by the policy.

The annuity provider (the insurer in particular) is exposed to several risk causes. In detail:

(I) Pricing and reserving for long-lasting products providing benefits in the case of survival call for appropriate projected life tables, that is, incorporating a forecast of future mortality trend. Anyway, future mortality patterns are of course affected by a significant degree of uncertainty. The longevity risk is then taken by the annuity provider.

(II) The long-lasting feature of the product combined with the minimum interest guarantee (commonly provided by the traditional, non-unit-linked products) implies the financial risk.

(III) The adverse selection risk is caused by the likely very good health conditions of individuals who purchase a life annuity, and hence by a presumably long expected lifetime.

Moving to the customer's perspective, we note what follows.

(1) The life annuity product relies on the mutuality mechanism. This means that:

   (a) the amounts released by the deceased annuitants are shared, as mortality credits, among the annuitants who are still alive;

   (b) on the annuitant's death, his/her estate is not credited with any amount, and hence no bequest is available.

(2) A traditional life annuity provides the annuitant with an "inflexible" post-retirement income, in the sense that the annual amounts must be in line with the benefit profile, as stated by the policy conditions.

(3) Purchasing a life annuity is an irreversible decision: surrendering is generally not allowed to annuitants (clearly, to avoid adverse selection effects). Hence, the life annuity constitutes an "illiquid" asset in the retiree's estate.

Features (1b), (2) and (3) can be perceived as disadvantages, and can hence weaken the propensity to immediately annuitize the whole amount available at retirement. These disadvantages can be mitigated, at least to some extent, either by purchasing life insurance products in which other benefits are packaged, or adopting specific annuitization strategies.

All the above features should carefully be considered by the insurer while planning the launch of a standard life annuity product.

2.2 Paths to innovation

The simple benefit structure underlying the standard SPIA can be generalized in various ways, according to different purposes. Possible generalizations are sketched in Fig. 1, ranging from traditional inclusion of riders (that is, supplementary benefits) to interesting innovations more recently proposed.

The most common traditional generalizations (see the box with blue frame in Fig. 1) are the following ones.

- A life annuity with *a guarantee period* (5 or 10 years, say), also named *period-certain life annuity*, pays the benefits over the guarantee period regardless of whether the annuitant is alive or not.

- The *money-back* (or *capital protection*) supplementary benefit consists in the addition of a death benefit to the life annuity product, then frequently called *value-protected life annuity*. In the case of early death of the annuitant, a value-protected annuity will pay to the annuitant's estate the difference (if positive) between the single premium and the cumulated benefits paid to the annuitant. Usually, capital protection expires at some given age (75, say), after which nothing is paid even if the above difference is positive.

- A *last-survivor annuity* is an annuity payable as long as at least one of two individuals (the annuitants), say (x) and (y), is alive. It can be stated that the annuity continues with the same annual benefit, say b, until the death of the last survivor. A modified form provides that the amount, initially set to b, will be reduced following the first death: to b' if individual (y) dies first, and to b'' if individual (x) dies first, usually with $b' < b$, $b'' < b$. Conversely, in many pension plans the last-survivor annuity provides that the annual benefit is reduced only if the retiree, say individual (x), dies first. Formally, $b' = b$ and $b'' < b$.

- An interesting rider is the *Long-Term Care (LTC) uplift*. In this case, a health-related benefit is added to the basic life annuity. The resulting product is a combination of a standard life annuity paid while the policyholder is healthy, and an uplifted income paid while the policyholder is claiming for the LTC benefit.

As noted in Sect. 2.1, a traditional life annuity provides the annuitant with an “inflexible” post-retirement income. Of course, the retiree can decide to obtain his/her post-retirement income withdrawing from a fund (that is, through a *drawdown* process), instead of purchasing a life annuity. Clearly, a drawdown process, while allows the choice of the periodic withdrawal amounts, leaves the retiree exposed to the risk of outliving his/her resources (the so called “individual longevity risk”). However, this risk can be hedged by purchasing a specific guarantee, named *Guaranteed Minimum Withdrawal Benefit (GMWB)*, offered by many insurers in the context of Variable Annuity products. This way, more flexibility is added to the post-retirement income profile (see the box with grey frame in Fig. 1).

Various linking arrangements can be conceived and implemented (see the box with orange frame in Fig. 1). Linking benefits to investment performance belongs to the insurance tradition: profit participation mechanisms are currently implemented in many insurance products (endowment insurance, whole-life insurance, life annuities). As regards life annuities, according to specific product designs a longevity-linking can also be introduced. This way, on the one hand part of the longevity risk can be transferred to the annuitants, and, on the other, the premium of the life annuity product can be conveniently reduced. This type of linking implies the definition of a *longevity-linked life annuity*. A longevity-linked life annuity involves a benefit adjustment process, according to which the annuity provider is entitled to reduce the benefit to all the annuitants in the event of an unanticipated increase in longevity. However, a floor amount should reasonably be stated to keep, at least to some extent, the guarantee characteristics which should feature all life annuity products. Of course, investment-linking and longevity-linking can be combined, so that possible decrease in the benefit because of unexpected mortality improvements can be offset by increase thanks to investment performance.

A standard life annuity is designed to provide a lifelong sequence of benefits from retirement age onwards. Nonetheless, restrictions to the age range covered by the benefits

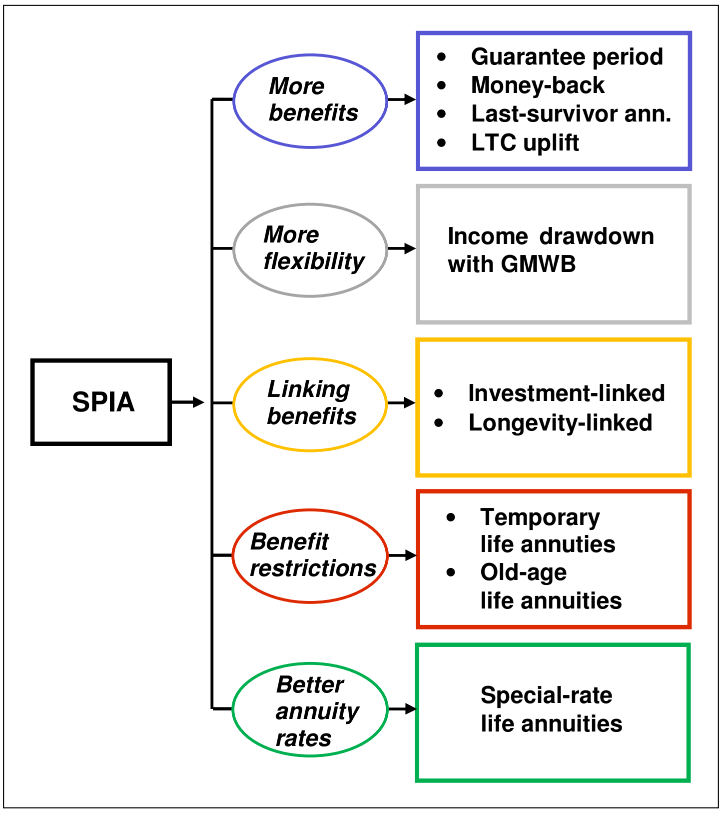


Figure 1: Generalizing the life annuity structure

can be implemented (see the box with dark red frame in Fig. 1). In particular, we consider the following product designs.

- A *temporary life annuity* pays to the annuitant a sequence of periodic benefits provided he/she is alive but up to a given age, 85 or 90 say, stated in the policy conditions (and hence at most for a given number of years). Hence, the tail of the lifetime distribution is not involved and the longevity risk borne by the annuity provider is consequently reduced. The premium of a temporary life annuity is of course lower than that of a standard life annuity. However, this type of product, in spite of the reduction in the premium, may be less attractive from the customer's perspective because of the "uncovered" age interval.

- *Old-age life annuities* (also known as *advanced life delayed annuities* [1]) can achieve a premium reduction effect thanks to a restriction in the opposite direction w.r.t. the temporary life annuity: an old-age life annuity pays a lifelong benefit but starting from a high age (75 or 80, say), and hence provides protection against the individual longevity risk. Of course, the annuity provider takes the exposure to the tail risk. A

[1] The advanced life delayed annuity (briefly, ALDA) has been proposed by Milevsky (2005).

(temporary) drawdown process, from retirement time to the commencement of the life annuity, will provide the retiree with post-retirement income.

As standard life annuities are attractive mainly for healthy people, in order to expand their business, in recent years some insurers have started offering better annuity rates to people whose health conditions are worse than those of standard annuity buyers. *Special-rate life annuity* products (see the box with green frame in Fig. 1) have then been designed. Special-rate life annuities are also called *underwritten life annuities*, because of the ascertainment of higher mortality assumptions via the underwriting requirements. Details on special-rate life annuities will be provided in Sect. 3.

2.3 Basic references

An extensive literature deals with technical and financial problems related to life annuities (and life insurance). As regards actuarial issues, see, for example, the following textbooks: Bowers et al. (1997), Dickson et al. (2013), and Olivieri and Pitacco (2015). A wide range of life insurance and life annuity products are described by Black and Skipper (2000).

Advantages and disadvantages of traditional life annuities are discussed by Milevsky (2005); the idea of old-age life annuities is then proposed, together with the relevant implementation, which leads to the ALDA (Advanced Life Delayed Annuity) product. On this issue, see also Gong and Webb (2010), and Stephenson (1978). Huang et al. (2009) generalize the idea of longevity insurance suggesting the design of an insurance product, i.e. the RCLA (Ruin Contingent Life Annuity), which generates a life annuity in the case of exhaustion of the (non-annuitized) fund because of poor investment performance or very long lifetime of the retiree.

The life annuity as a solution (from the individual perspective) to the individual longevity risk is widely discussed by Wadsworth et al. (2001) and Swiss RE (2007). A survey of annuity pricing is provided by Cannon and Tonks (2006).

Guarantees in life annuities (and life insurance) products are addressed, in particular, by Gatzert (2009), Hardy (2004) and Pitacco (2012). The paper by Boyle and Hardy (2003) focuses on the impact of the GAO, that is, the Guaranteed Annuity Option. Guarantee structures in life annuities constitute the main topic of the paper by Pitacco (2016).

Variable annuities are investment products which, thanks to several options that can be exercised by the policyholder, can provide several guarantees involving the accumulation period as well as the post-retirement period (for example the GMWB). The interested reader can refer, for example, to Kalberer and Ravindran (2009) and Ledlie et al. (2008).

In the more general framework of post-retirement income, we first cite the books by Milevsky (2006, 2013), which provide an in-depth analysis of possible choices for the income construction and the specific role of life annuities in this context; the interested reader should in particular refer to Milevsky (2013) for a detailed literature review. Several post-retirement products are described by Rocha et al. (2011). Shapiro (2010), while discussing post-retirement financial strategies, also provides an extensive literature review.

Life annuities under a historical perspective are addressed by Kopf (1926), Poterba (1997), Milevsky (2013) and, in the framework of actuarial science, by Haberman (1996).

Longevity-linked life annuities are widely discussed in the actuarial literature. We cite some recent contributions which should be considered by the interested reader: Denuit et al. (2011), Goldsticker (2007), Maurer et al. (2013), Lüty et al. (2001), Olivieri and Pitacco (2020a,b), Piggott et al. (2005), Richter and Weber (2011), Sherris and Qiao (2013) and van de Ven and Weale (2008).

References regarding special-rate life annuities (and risk classification) will be given in Sect. 3.4.

3 Special-rate life annuities

Special-rate life annuities constitute the main topic of this research. General issues are described in this section, paving the way to the risk assessments that will be presented and discussed in Sects. 7, 8 and 9.

3.1 Purposes

As noted in Sect. 2.2, standard life annuities are attractive mainly for healthy people. Premium rates are consequently kept high. Hence, a large portion of potential annuitants are out of reach of insurers (see Fig. 2). At the same time, individuals in (more or less) poor health conditions are excluded from lifelong annuity benefits.

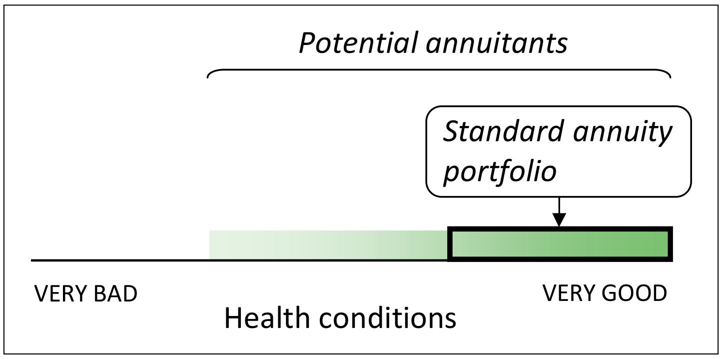


Figure 2: Potential annuitants population and standard annuity portfolio

Lowering the premium rate would of course raise the attractiveness of the life annuity product (see Fig. 3), but would result in a more heterogeneous portfolio and in a likely underpricing in particular for healthy annuitants. Such a solution should then be considered unrealistic.

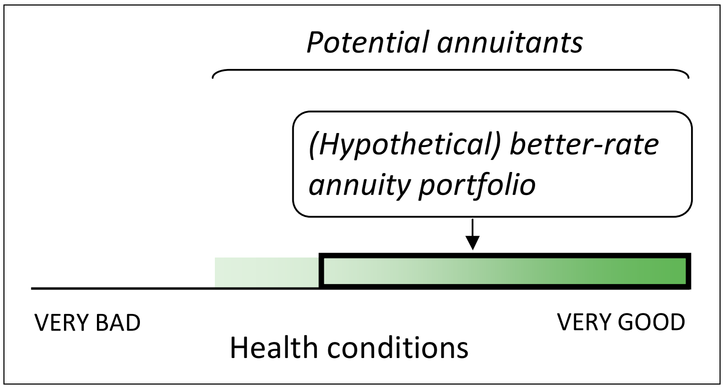


Figure 3: Potential annuitants population and (unrealistic) better-rate annuity portfolio

In order to expand their business, some insurers have recently started offering better annuity rates to people whose health conditions are worse than those of likely buyers of standard life annuities (see Fig. 4). Special-rate life annuity products have then been designed and proposed.

Standard life annuities are commonly sold without any underwriting. Conversely, underwriting procedures are required for special-rate annuities, in order to adopt appropriate mortality assumptions. For this reason, special-rate annuities are also called, as already noted, underwritten annuities.

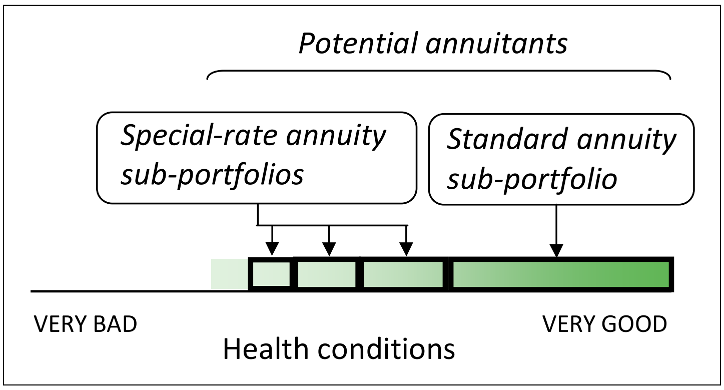


Figure 4: Potential annuitants population and annuity portfolio also consisting of three special-rate annuity subportfolios

3.2 Underwriting schemes

Underwriting for special-rate life annuities can be implemented in a number of ways, and several classifications can be conceived. [2]

It is interesting to focus on:

(1) what risk factors can be chosen as rating (or "pricing") factors, besides annuitant's age and gender (if permitted by the local current legislation);

(2) how many rating factors are actually accounted for in the underwriting process of a given special-rate annuity;

(3) how many rating classes, that is, how many different annuity rates, are defined.

As regards (1), we note that higher mortality, and then lower life expectancy, may generally be due to the following causes.

[2] What follows is mainly based on the classifications proposed by Rinke (2002). Differences in terminology can however be detected. In particular, the expression "enhanced annuity" is used, in that paper, as a synonym for underwritten or special-rate life annuity.

(1a) The individual *health*, and in particular the presence of some past or current *disease*, clearly impacts on the mortality pattern.

(1b) The applicant's *lifestyle* (e.g. smoking and drinking habits, sedentary life, etc.) can cause higher mortality.

(1c) The *environment* in which the applicant lives might also impact on his/her mortality, and hence socio-geographical risk factors can be accounted for.

On the one hand, the higher the number of rating factors (see point (2) above), the more complex is the underwriting process; on the other, the higher the number of rating classes (see point (3)) the better is the fitting of the individual risk profile.

As regards the number of rating classes, the following classification reflects alternative pricing schemes that can be adopted in the insurance practice.

(3a) When a *single-class* underwriting scheme is adopted, one or just a few rating factors are accounted for, and the underwriting results in a yes/no answer. If yes, a given annuity rate, higher than the "standard" one, is applied. An example is given by smoking habits. The portfolio then consists of a standard annuity sub-portfolio and a special-rate annuity sub-portfolio.

(3b) The *multi-class* underwriting scheme can be implemented either considering just one rating factor with several possible values, or more rating factors in which case each combination of values yields an annuity rate. More than one special-rate sub-portfolio is the result of this scheme.

(3c) The *individual underwriting* allows to use all the available information about the individual, so that the annuity rate can be tailored on the applicant's characteristics. Also this approach will result in diverse special-rate sub-portfolios.

The above classifications are sketched in Fig. 5.

3.3 Taxonomy of special-rate annuities

The following types of special-rate annuities are sold in several markets. For a given amount of single premium, the benefit amount depends on the annuity type.[3]

1. A *lifestyle annuity* pays out a benefit higher than a standard annuity because of factors (e.g. smoking and drinking habits, marital status, occupation, height and weight, blood pressure and cholesterol levels) which might result, to some extent, in a shorter life expectancy.

[3] We recall that the terminology is not univocally defined; see also Note 2.

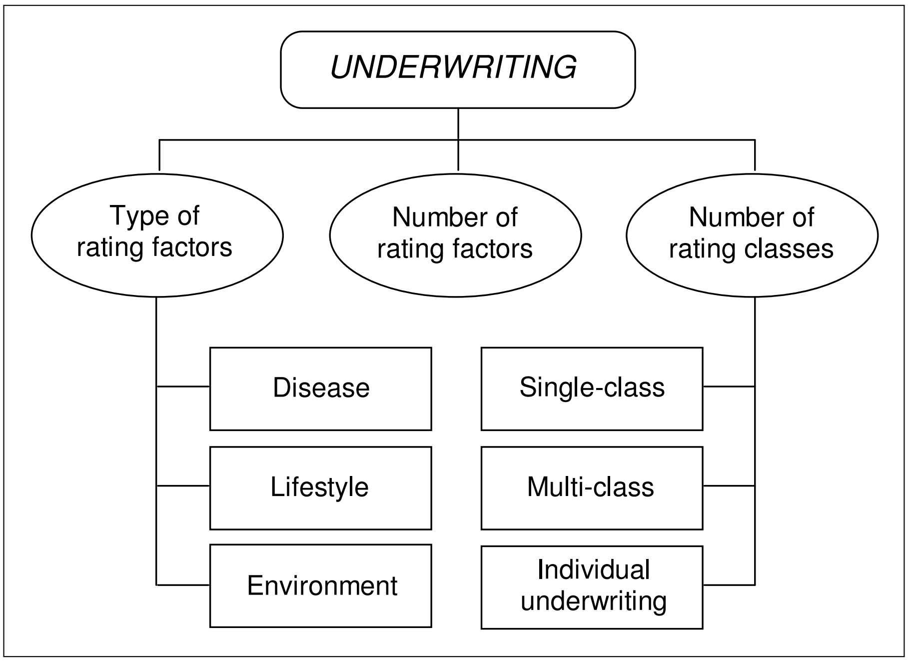


Figure 5: Approaches to underwriting for special-rate life annuities

2. The *enhanced life annuity* pays out benefits to a person with a reduced life expectancy, in particular because of a personal history of medical conditions. Of course, the "enhancement" in the annuity benefit (compared to a standard-rate life annuity, same premium) comes, in particular for this type of annuity, from the use of a higher mortality assumption.

3. The *impaired life annuity* pays out higher benefits than an enhanced life annuity, as a result of medical conditions which significantly shorten the life expectancy of the annuitant (e.g., diabetes, chronic asthma, cancer, etc.).

4. Finally, *care annuities* are aimed at individuals with very serious impairments or individuals who are already in a senescent-disability (or long-term care) state.

Thus, moving from type 1 to type 4 results in progressively higher mortality assumptions, shorter life expectancy, and hence, for a given single premium amount, in higher annuity benefits.

Regarding the underwriting requirements, we note what follows.

The underwriting of a lifestyle annuity can take into account one or more rating factors, and can result in a single-class or a multi-class underwriting. Examples are given, as already noted, by smoking and drinking habits, marital status, and occupation. These factors might result in a shorter life expectancy. Some specific examples follow.

1(a) *Smoker annuities*: if the applicant has smoked at least a given number of cigarettes for a certain number of years, he/she is eligible for a smoker annuity. A single-class underwriting is in this case implemented.

1(b) Mortality differences between married and unmarried individuals underpin the use of special rates in pricing the *unmarried lives annuities*. The observed higher mortality rates of unmarried individuals justify a higher annuity rate. A single-class underwriting is also in this case implemented.

The applicant's health status and, in particular, the presence of past or current diseases is considered in the special-rate annuities of types 2, 3 and 4. Various factors concerning the health status can be accounted for, and this usually leads to a multi-class underwriting. Medical ascertainment is of course required. In particular, as regards the impaired life annuity and the care annuity, the underwriting process must result in classifying the applicant as a *substandard risk*. For this reason, these annuities are sometimes named *substandard annuities*.

The above list of special-rate annuity types can be completed by the *postcode annuities*, which constitute an interesting example of environment-based rating (see 1(c) in Sect. 3.2). The postcode can provide a proxy for social class and location of housing, i.e. risk factors which have a significant impact on the life expectancy. Then, its use as a rating factor for pricing life annuities can be justified. Hence, a multi-class underwriting scheme follows.

3.4 Basic references

Special-rate life annuities are described in various papers and technical reports: see, in particular Ainslie (2000), Drinkwater et al. (2006), Ridsdale (2012), Rinke (2002) and Weinert (2006). The article by Edwards (2008) is specifically devoted to life annuity rating based on the postcode (that is, a proxy for social class and location of housing). Socio-geographic variations in mortality are analyzed, for example, by Howse et al. (2011).

Risk classification in life insurance and life annuities is addressed in many books and papers; a compact review, together with an extensive reference list, is provided by Haberman and Olivieri (2014). The impact of risk classification on the structure of life annuity portfolios is dealt with by Gatzert et al. (2012), Hoermann and Russ (2008) and Olivieri and Pitacco (2016). An extensive literature focuses on the impact of heterogeneity due to unobservable risk factors, usually summarized by the individual “frailty”, on the results of a life annuity portfolio. For a detailed bibliography, the reader can refer to Pitacco (2019), where relations between mortality at high ages and frailty are also addressed.

4 Market issues

We first address supply and demand of special-rate annuities. We then focus on extra-mortality which can entitle to better annuity rates. Practical aspects of pricing special-rate annuities conclude this section.

4.1 Supply and demand

In what follows, we first focus on the UK market, from which interesting insights into various aspects of the special-rate annuity business can be obtained. Conversely, according to experts' opinions, other markets and, notably, the US and the Canada markets are still small if compared to the potential numbers of people entitled to purchase special-rate annuities.

In the UK market special-rate annuities constitute a significant portion of the life annuity business. The UK market of special-rate annuities started in the mid-1990s with the "informal" proposal from two insurers. In 1995 the first "official" special-rate annuities were launched. The development of that market is sketched in Fig. 6.

Regarding the demand for special-rate annuities in UK (see Fig. 7, where special-rate annuities are denoted "enhanced annuities"), we note what follows. Prior to April 2014, retirees were required by law to annuitize resources accumulated in their individual defined contribution pension plan. After that date, retirees had a choice: either purchase a life annuity, or taking the accumulated amount in cash, or investing the amount and then obtain the post-retirement income via a drawdown process. This possibility had a significant impact on the life annuity market and on the sales of special-rate annuities in particular. However, it is worth noting that the share of special-rate annuities (w.r.t. the total amount of life annuity business) remained constant.

Turning to general aspects, non country-related, it is interesting to analyze barriers on the supply side and the demand side.

Regarding the supply side, we first note that the launch of special-rate annuities can impact on the standard life annuity portfolio. In particular:

- future sales of standard annuities might decrease because of a shift of eligible customers towards special-rate annuities (the so called "cannibalization" effect);
- as a consequence, annuitants' mortality in the standard annuity portfolio might decrease, hence requiring a revision of the biometric assumptions underlying standard premium rates.

Further supply-side barriers are related to pricing difficulties because of uncertainty

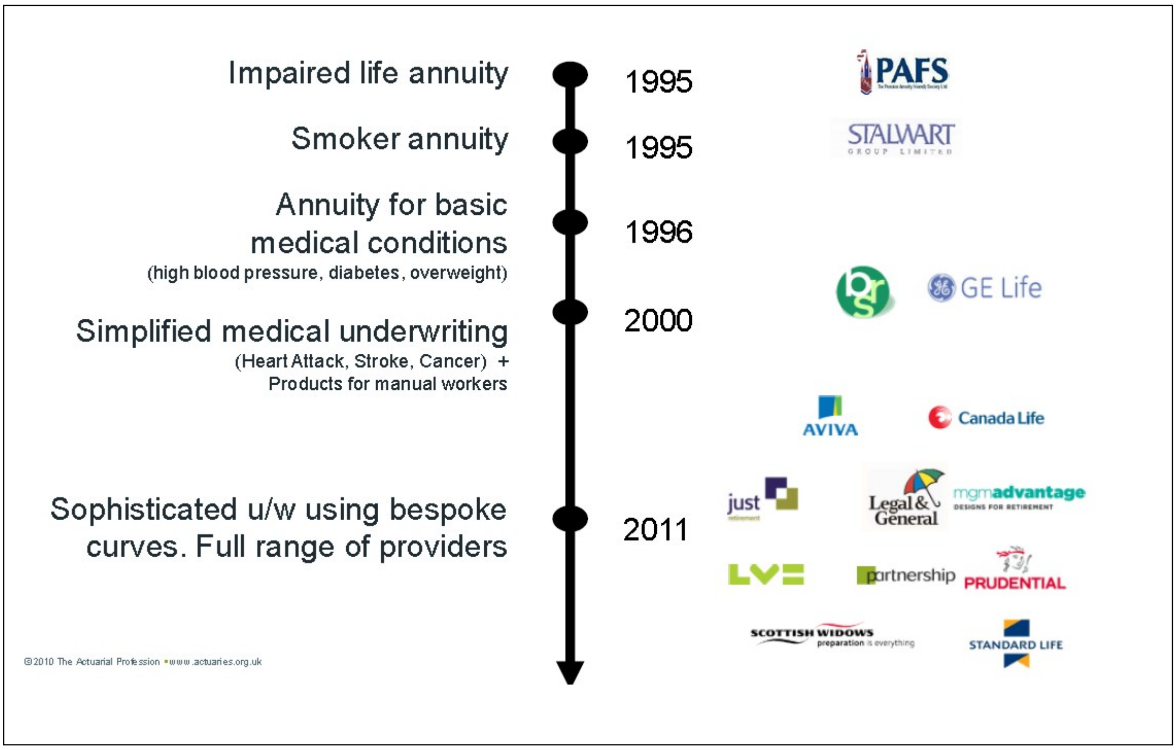


Figure 6: Development of special-rate annuity supply in the UK [Source: Moyle et al. (2011)]

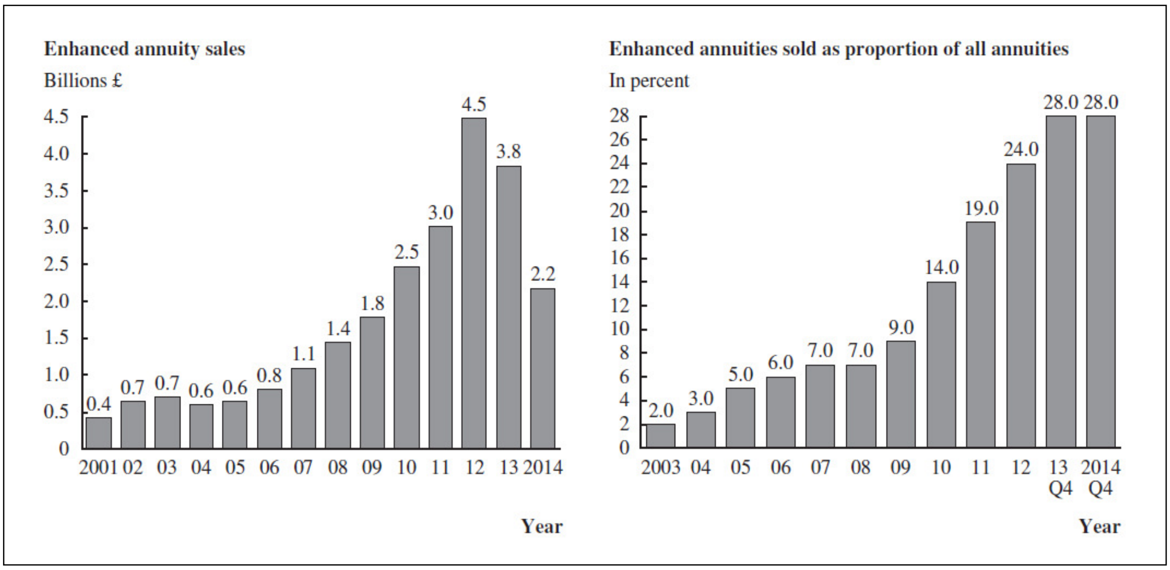


Figure 7: The demand for special-rate annuities in the UK market [Source: Gatzert and Klotzki (2016); data from Towers Watson and Association of British Insurers]

in mortality data and difficulties in implementing an effective underwriting system (and the consequent risk of overestimating the impact of lifestyles, and of past and current diseases).

As regards the demand side, a barrier to the purchase of life annuities in general and of special-rate annuities in particular is due, on the one hand, to the poor financial literacy of many potential customers and to the preference for managing funds through drawdown, and, on the other, to the absence of clear perception of the individual longevity risk (so originating the well known "annuity puzzle"). In this regard, see also Sect. 2.1.

4.2 Statistical data

Detailed data on the mortality of special-rate annuitants are not publicly available. Conversely, mortality data related to diverse lifestyles or pathologies can be found in many papers and technical reports.

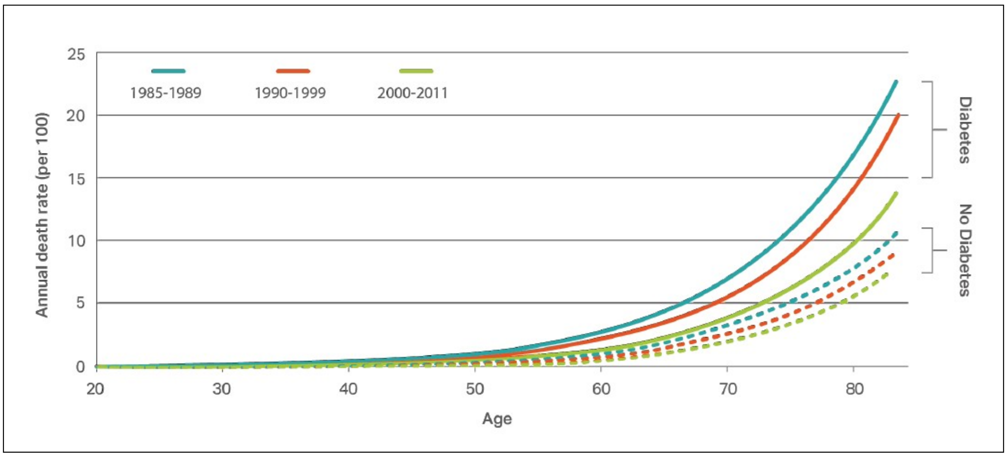


Figure 8: All-cause annual death rates among men, US, by age, for diabetics and non-diabetics [Source: PartnerRe (2020)]

Examples relate to:

- all-cause mortality of diabetics versus non-diabetics (see mortality rates displayed in Fig. 8, and survival functions in Fig. 9);
- all-cause mortality of smokers versus non-smokers (represented in terms of survival functions in Fig. 10).

It is worth noting that survival functions in Fig. 9 show a higher dispersion of the lifetime distribution for diabetics w.r.t. non-diabetics, as witnessed by a weaker "rectangularization" of the curves.

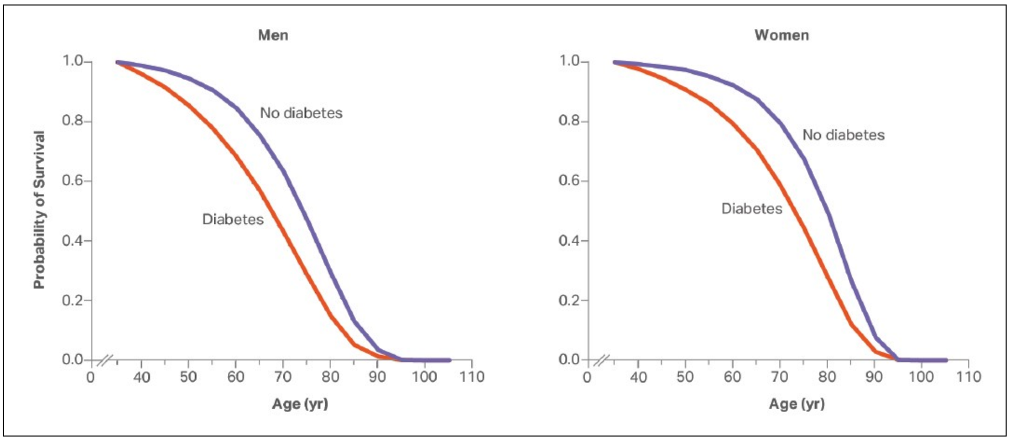


Figure 9: Survival functions for diabetic and non-diabetic people, US [Source: PartnerRe (2020)]

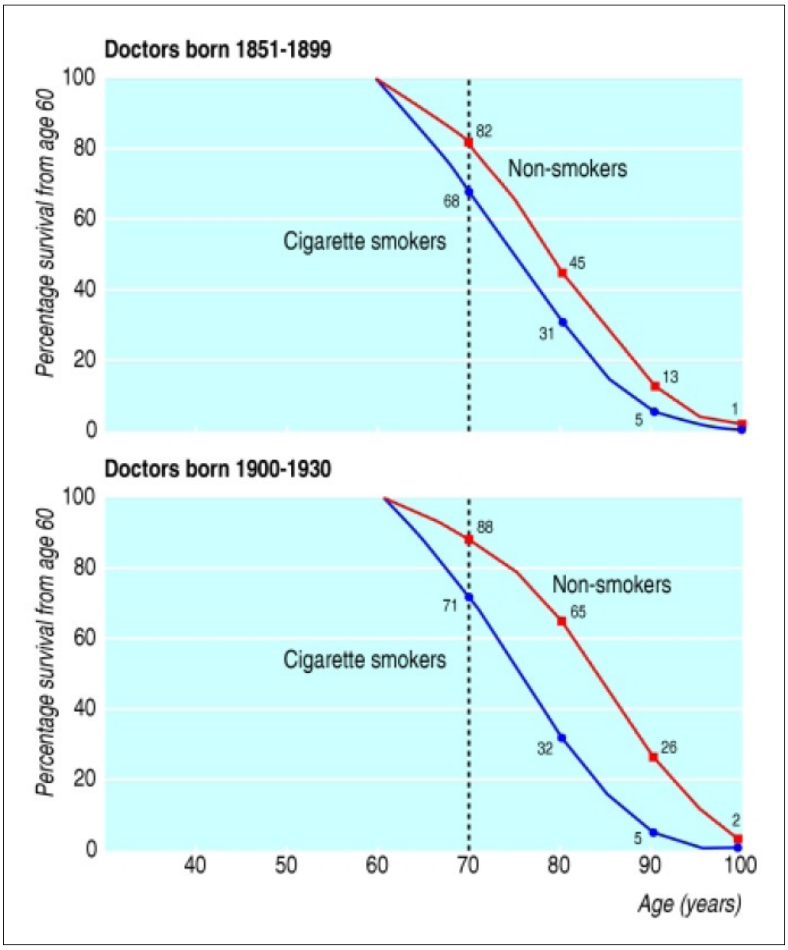


Figure 10: Survival functions (from age 60) for smokers and non-smokers [Source: Doll et al. (2004)]

An interesting example of age-distributions of deaths of individuals receiving benefits from standard annuities, enhanced annuities and impaired annuities respectively is given in Fig. 11. The increasing dispersion in the distributions, moving from standard to enhanced and to impaired annuities, is apparent. The higher dispersion is likely attributable to uncertainty in cause of death for individuals with one in a broad range of pathologies.

Of course, a lower dispersion could be obtained by restricting to a small number of pathologies the eligibility for a special-rate annuity.

The case-studies, whose results will be presented in Sects. 7 and 8, are based on distribution assumptions similar to those sketched in Fig. 11.

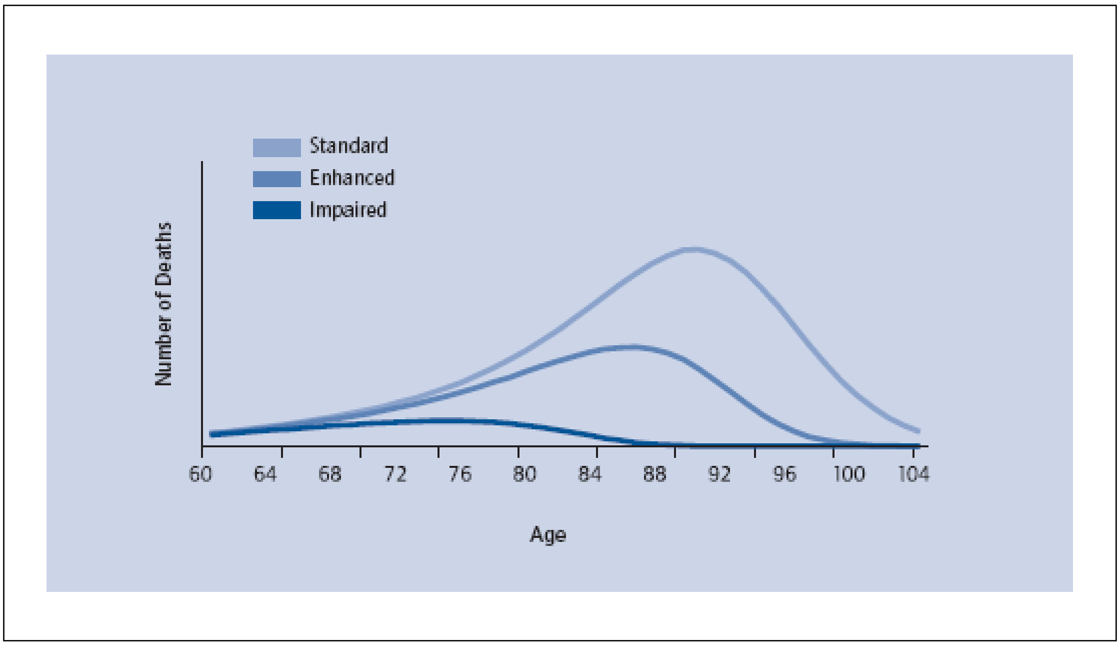


Figure 11: Age-distribution of deaths for males age 60 [Source: Weinert (2006)]

4.3 Pricing practice

Specific biometric assumptions must be adopted for pricing special-rate annuities, in order to reflect the diverse age-patterns of mortality corresponding to different lifestyles, pre-existent pathologies, current pathologies, etc.

An example, referred to the UK market, of how different lifestyles (smoking in particular) and pathologies impact on the annuity benefit (for a given amount of single premium) is sketched in Fig. 12.

Moving to general issues, we note that two approaches can be adopted to determine special rates and hence special-annuity premiums:

- use of tables of specific mortality rates (or mortality laws) constructed by observing mortality of individuals affected by specific pathologies (e.g., diabetes, stroke, etc.);
- choice of adjustment models which modify (increase) the mortality rates provided by a standard life table.

While the former approach is not very common because of difficulties in constructing reliable tables, the latter is frequently adopted as it only requires the estimation of some parameters (see the following formulae). A similar approach is usually adopted also in life insurance to assess substandard risks.

Although this topic is outside the scope of the numerical assessments which are described and commented in Sects. 7, 8 and 9, we provide a brief description of possible adjustment models.

Figure 12: Increase in the annuity benefit (on a monthly basis) for 65 year old with a 50,000 GBP fund (single premium), no guarantee period [Source: Sharingpensions.co.uk research: UK insurance companies]

A (rather) general adjustment model can be defined as follows.

$$q_{x,t}^{(h)} = A_{x,t}^{(h)}\, q_{x+s^{(h)}+t} + B_{x,t}^{(h)} \tag{1}$$

where:

x is the age at policy issue, while $x+t$ is the current age;

q denotes the standard mortality rates (adopted for standard annuities);

h denotes a rating class, and $q^{(h)}$ the related mortality rate;

$A_{x,t}^{(h)}$ is the multiplicative adjustment factor;

$B_{x,t}^{(h)}$ is the additive adjustment factor;

s is the “years-to-age” addition, also called “age-shift” parameter.

The following models constitute particular implementations of model 1.

- Multiplicative model:
$$q_{x,t}^{(h)} = A_{x,t}^{(h)}\, q_{x+t} \tag{2}$$
- Additive model:
$$q_{x,t}^{(h)} = q_{x+t} + B_{x,t}^{(h)} \tag{3}$$
- Years-to-age addition model:
$$q_{x,t}^{(h)} = q_{x+s^{(h)}+t} \tag{4}$$

Further, by assuming the parameters A and B independent of both age x and past duration t, we obtain, from Eqs. (2) and (3) respectively, formulae frequently used, together with (4), in the actuarial practice:

- constant percentage extra-mortality:
$$q_{x,t}^{(h)} = A^{(h)}\, q_{x+t} \tag{5}$$
- flat extra-mortality:
$$q_{x,t}^{(h)} = q_{x+t} + B^{(h)} \tag{6}$$

The choice of the model and the estimation of the relevant parameters must be driven by the specific pathology (or set of pathologies).

4.4 Basic references

An interesting analysis of market issues related to special-rate annuities is presented by Gatzert and Klotzki (2016), where barriers on the supply side and the demand side are in particular addressed.

Statistics regarding extra-mortality by various causes are reported and commented in a number of papers and technical reports. Referring to the topics addressed in Sect. 4.2, see for example: Doll et al. (2004), Ebrahim et al. (1985), Laing et al. (1999), Levy et al. (2002), Ou et al. (2016), PartnerRe (2020), Soedamah-Muthu et al. (2006), Swerdlow and Jones (1996). Impact of disabilities on the life expectancy is analyzed by Thomas and Barnes (2010).

Practical pricing aspects are dealt with by Gracie and Makin (2006) and James (2016).

Adjustment models which can be used to represent specific mortality rates are described in Pitacco (2019).

The design and the launch of special-rate annuities are described, in the framework of the "new product development" process, in Chap. 9 of Pitacco (2020) (see also the relevant bibliographic references).

5 The biometric model

We present the biometric assumptions we have adopted to implement the actuarial model and the risk profile assessments.

5.1 The basic assumption

To represent the age-patterns of mortality, we have adopted the well known Gompertz law. The force of mortality is then given by the following expression:

$$\mu_x = B\,c^x, \quad \text{with } B, c > 0 \tag{7}$$

Instead of using the standard parametrization (7), we refer to the "informative" parametrization (see, for example, Carriere (1992)), that is:

$$\mu_x = \frac{1}{D}\exp\Big(\frac{x-M}{D}\Big), \quad \text{with } M, D > 0 \tag{8}$$

where M denotes the mode of the Gompertz probability density function and D a measure of dispersion.

The survival function is given by:

$$S(x) = \exp\Big(\exp\Big(-\frac{M}{D}\Big) - \exp\Big(\frac{x-M}{D}\Big)\Big) \tag{9}$$

The relations with the traditional parameters are as follows:

$$c = \exp\Big(\frac{1}{D}\Big) \tag{10}$$

$$B = \frac{\exp\Big(-\dfrac{M}{D}\Big)}{D} \tag{11}$$

5.2 Specific mortality assumptions

The parametrization expressed by Eq. (8) is notably useful when making appropriate choices regarding the location and the dispersion of the lifetime distribution.

Following the suggestions expressed by Fig. 11, we will set the parameters M and D to obtain diverse probability density functions as represented in Fig. 13 (see Sect. 7.1).

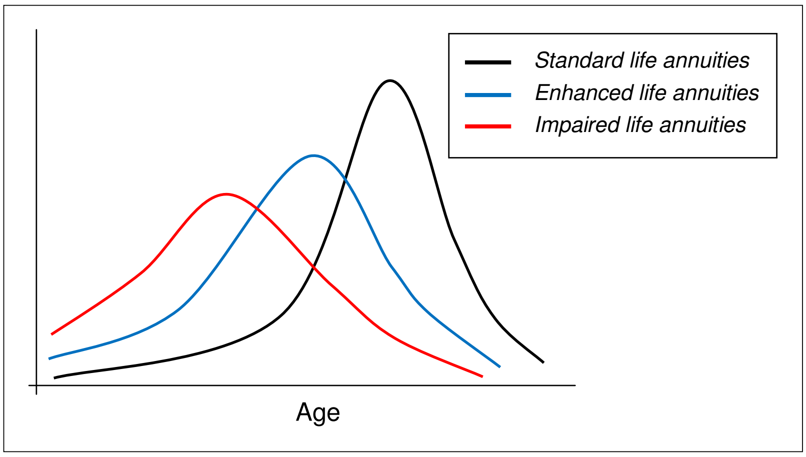


Figure 13: Curves of deaths for different life annuity sub-portfolios

5.3 Basic references

Biometric functions and, in particular, mortality laws are described in all the actuarial textbooks. See, for example, Olivieri and Pitacco (2015) and references therein. The "informative" parametrization of the Gompertz law has been proposed by Carriere (1992).

6 The actuarial model

After defining the portfolio structures used in the various evaluations, we define the quantities referred to in the deterministic assessments and in the stochastic assessments.

6.1 Portfolio structures

We will consider a life annuity portfolio P generally consisting of three subportfolios:

- subportfolio $\mathrm{SP1}$ initially consisting of n_1 standard life annuities;
- subportfolio $\mathrm{SP2}$ initially consisting of n_2 enhanced life annuities;
- subportfolio $\mathrm{SP3}$ initially consisting of n_3 impaired life annuities.

Of course, one of the n_k can be set equal to 0. Let n denote the size of the portfolio P, that is:

$$n = n_1 + n_2 + n_3$$

Assumptions underlying the actuarial model are as follows.

- The lifetime distribution for annuitants in the subportfolio $\mathrm{SP}k$ follows the Gompertz law with parameters M_k and D_k, $k = 1, 2, 3$.
- All the annuitants are age x at policy issue.
- We assume independence among the individual lifetimes in each subportfolio and in the portfolio P.
- The same benefit b is paid by all the life annuity policies.

6.2 Actuarial values

Our ultimate object is to analyze the behavior of various quantities defined as functions of n_1, n_2, n_3, in particular: expected value, variance and coefficient of variation (risk index) of the portfolio payouts.

To this purpose, we first recall the basic formulae referred to an immediate life annuity, with benefit $b = 1$ paid in arrears to an individual age x at policy issue.

The expected present value of the annual benefits is given (according to the traditional actuarial notation) by:

$$a_x = \sum_{h=1}^{\omega-x} a_{\overline{h}|} \; {}_{h|1}q_x \tag{12}$$

where:

- ω denotes the maximum attainable age;
- the present value $a_{\overline{h|}}$ of an annuity-certain is given by:

$$a_{\overline{h|}} = \frac{1-(1+i)^{-h}}{i} \tag{13}$$

with i denoting the interest rate used for discounting;

- the probability of a person age x of dying between $x+h$ and $x+h+1$, ${}_{h|1}q_x$, is given by:

$${}_{h|1}q_x = \frac{S(x+h)-S(x+h+1)}{S(x)} \tag{14}$$

of course, specific survival functions S will be used for the various subportfolios.

The variance of the present value of the annual benefits is given by:

$$\sigma_x^2 = \sum_{h=1}^{\omega-x} a_{\overline{h|}}^2 \, {}_{h|1}q_x - (a_x)^2 \tag{15}$$

We denote with $E_k(n_k)$ and $V_k(n_k)$ the expected value and the variance of the benefit payouts of subportfolio $\mathrm{SP}k$. Assuming a benefit $b=1$, we obviously have:

$$E_k(n_k) = n_k \, a_x; \quad k=1,2,3 \tag{16}$$

and, thanks to the assumption of independence among the individual lifetimes:

$$V_k(n_k) = n_k \, \sigma_x^2; \quad k=1,2,3 \tag{17}$$

For a generic portfolio P, consisting of $n = n_1+n_2+n_3$ policies, we then find:

$$E(n_1,n_2,n_3) = \sum_{k=1}^{3} E_k(n_k) \tag{18}$$

$$V(n_1,n_2,n_3) = \sum_{k=1}^{3} V_k(n_k) \tag{19}$$

6.3 The risk index

The risk index (or coefficient of variation) is, as well known, a relative risk measure that expresses the variability of a random quantity in terms of standard deviation per unit of expected value.

It is a risk measure frequently adopted in risk theory and risk management to assess the so called pooling effect, that is, the diversification effect achieved by constructing a pool of risks.

For a generic portfolio P, the risk index ρ is defined as follows:

$$\rho(n_1, n_2, n_3) = \frac{\sqrt{V(n_1, n_2, n_3)}}{E(n_1, n_2, n_3)} \tag{20}$$

We note that ρ is unit-free and, in particular, independent of the benefit amount b.

6.4 Cash flows and portfolio fund

Annual cash flows are, of course, random quantities. For the generic subportfolio $\mathrm{SP}k$, the random cash flow (payout) at time t, $CF_k(t)$, depends on the number $N_k(t)$ of annuitants alive at that time, and is of course given by:

$$CF_k(t) = b\, N_k(t); \quad k = 1, 2, 3 \tag{21}$$

Referring to a generic portfolio P, consisting of three subportfolios, the random cash flow at time t is then given by:

$$CF(t) = \sum_{k=1}^{3} CF_k(t) \tag{22}$$

The portfolio fund is generally defined as the accumulated value of all the portfolio cash flows, that is:

- the total amount of single premiums;
- the annual benefit payouts;
- the shareholders' capital allocations and releases.

We are only interested in the analysis of the portfolio risk profile (and not in the analysis of portfolio profits). Then, for a portfolio of annuities paying the benefit b:

- we assume a total amount of equivalence net single premiums, Π, given by

$$\Pi = b\, E(n_1, n_2, n_3) \tag{23}$$

- the fund accumulation works at interest rate i, that is, the same rate used for calculating present values;

- we disregard shareholder's capital allocation / releases.

Hence, the (random) portfolio fund at time t, denoted by $F(t)$ is given by the following expression:

$$F(t) = \Pi(1+i)^t - \sum_{h=1}^{t} CF(t)\,(1+i)^{t-h} \tag{24}$$

6.5 Basic references

Basic actuarial quantities (actuarial values, risk index, portfolio fund, etc.) are defined in all the actuarial textbooks. See, for example, Olivieri and Pitacco (2015) and references therein.

7 Assessments of portfolios' risk profile: Deterministic approach

In this section we analyze the impact of:

- the portfolio structure (see Sect. 7.2)
- the lifetime distributions (see Sect. 7.3)

on the risk profile expressed in terms of the risk index.

Each set of "cases" in Sects. 7.2 and 7.3 consists of a set of diverse portfolio structures, denoted by P01, P02, etc. Comments on the numerical results can be found at the end of Sects. 7.2 and 7.3 respectively.

7.1 Biometric assumptions

As noted in Sects. 5.1 and 5.2, we have adopted the Gompertz law, with specific parameters to represent diverse lifetime distributions for standard life annuities ($k = 1$), enhanced annuities ($k = 2$), and impaired annuities ($k = 3$).

The baseline assumptions for the parameters M_k and D_k are represented in Table 1. Other assumptions will be adopted to perform a sensitivity analysis (see Sect. 7.3).

Table 1: Parameters of the Gompertz law

k	M_k	D_k
1	90	5
2	80	8
3	70	13

The functions d_x, l_x and q_x are plotted in Figs. 14, 15 and 16,

$$q_x = \frac{S(x) - S(x+1)}{S(x)} \tag{25}$$

$$\ell_x = 100\,000\, S(x) \quad \text{(survival curve)} \tag{26}$$

$$d_x = \ell_x - \ell_{x+1} \quad \text{(curve of deaths)} \tag{27}$$

Of course, all the quantities in the above equations must specifically be referred to the diverse mortality assumptions k ($k = 1, 2, 3$). The notation $\mathrm{LT}k$ is used to refer to the life table corresponding to the mortality assumption k.

Figure 14: Life table comparison: d_x

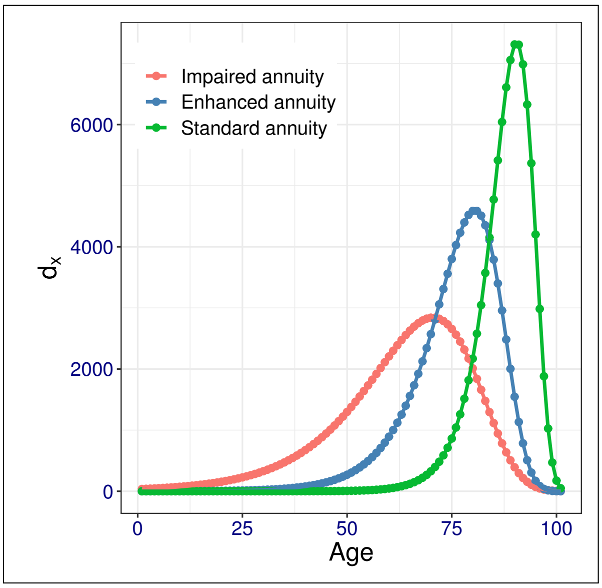
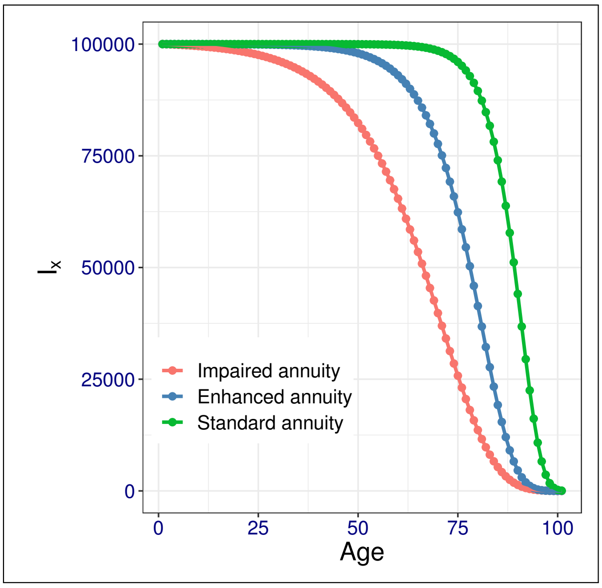


Figure 15: Life table comparison: l_x



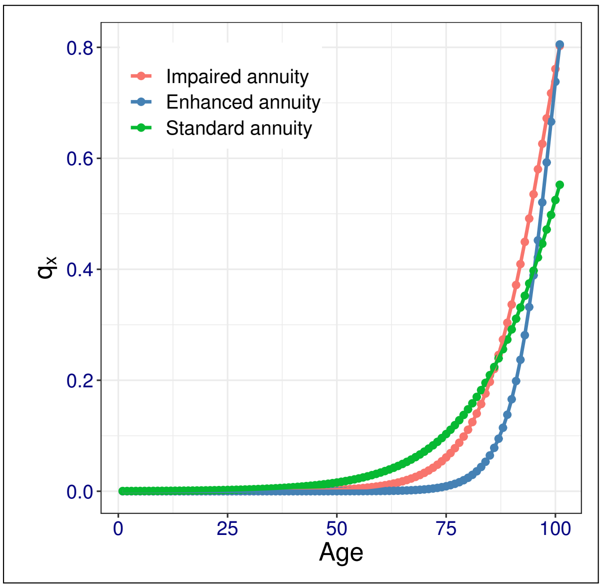


Figure 16: Life table comparison: q_x

7.2 Impact of the portfolio structure

In this section we analyze the impact of diverse portfolio structures on the portfolio risk profile. Portfolio structures are defined in terms of the subportfolio sizes n_1, n_2, n_3. The risk profile is summarized by the risk index (or coefficient of variation) of the present value of the total portfolio payout (see Eq. (20)).

Biometric assumptions are as specified in Table 1, if not otherwise stated.

Cases 1.1

We analyze the impact of the size of the subportfolio $\mathrm{SP2}$ of enhanced annuities. Then:

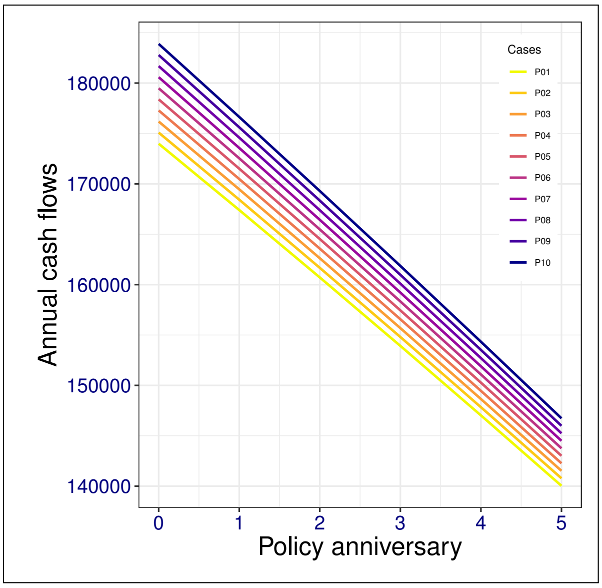
$$n_1 = 10\,000$$
$$n_2 = 100, 200, \ldots, 1\,000$$
$$n_3 = 0$$

Results are shown in Table 2.

Table 2: Cases 1.1 - impact of the portfolio structure on the risk index

Portfolio	n_2	$\rho(10\,000, n_2, 0)$
P01	100	0.002378268
P02	200	0.002381505
P03	300	0.002384564
P04	400	0.002387452
P05	500	0.002390176
P06	600	0.002392740
P07	700	0.002395152
P08	800	0.002397415
P09	900	0.002399535
P10	$1\,000$	0.002401517

Figure 17: Cases 1.1 - impact of the portfolio structure on the annual cash flows



Cases 1.2

We analyze the impact of the size of the subportfolio $\mathrm{SP3}$ of impaired annuities. Then:

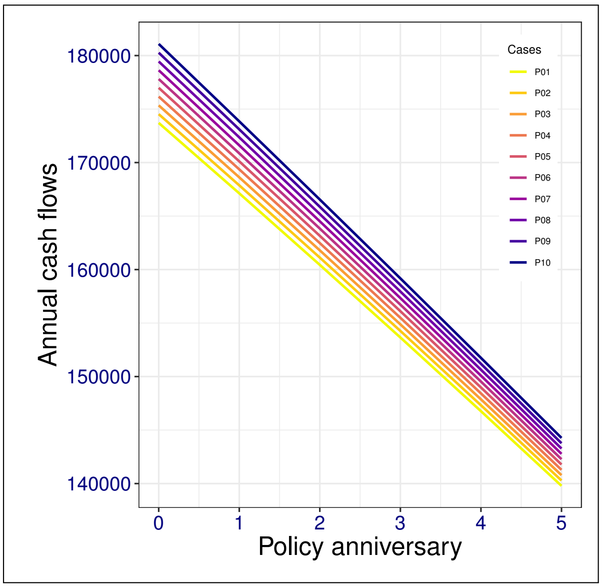
$$n_1 = 10\,000$$
$$n_2 = 0$$
$$n_3 = 100, 200, \ldots, 1\,000$$

Results are shown in Table 3.

Table 3: Cases 1.2 - impact of the portfolio structure on the risk index

Portfolio	n_3	$\rho(10\,000, 0, n_3)$
P01	100	0.002382799
P02	200	0.002390524
P03	300	0.002398028
P04	400	0.002405318
P05	500	0.002412401
P06	600	0.002419283
P07	700	0.002425969
P08	800	0.002432465
P09	900	0.002438776
P10	$1\,000$	0.002444908

Figure 18: Cases 1.2 - impact of the portfolio structure on the annual cash flows



Cases 1.3

We assume that both enhanced annuities and impaired annuities are launched (together with standard annuities), and analyze the joint impact by assuming that $n_3 = n_2/2$. Then:

$$n_1 = 10\,000$$
$$n_2 = 500, 600, \dots, 1\,000$$
$$n_3 = 250, 300, \dots, 500$$

Results are shown in Table 4.

Table 4: Cases 1.3 - impact of the portfolio structure on the risk index

Portfolio	n_2	n_3	$\rho(10\,000, n_2, n_3)$
P01	500	250	0.002407197
P02	600	300	0.002412496
P03	700	350	0.002417448
P04	800	400	0.002422070
P05	900	450	0.002426375
P06	$1\,000$	500	0.002430381

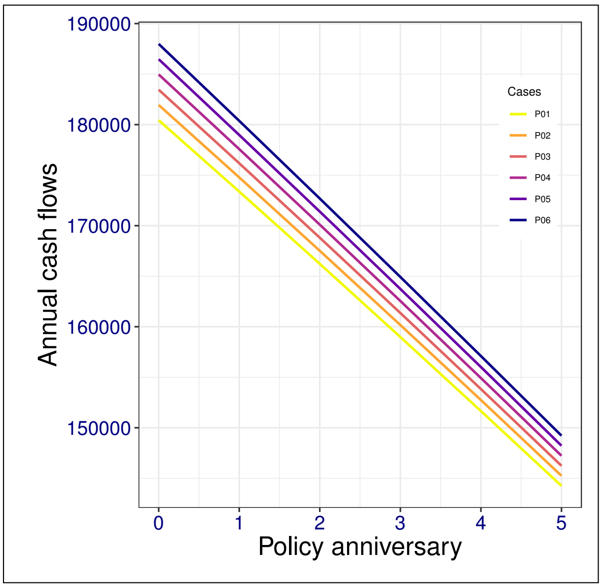


Figure 19: Cases 1.3 - impact of the portfolio structure on the annual cash flows

Cases 1.4

As noted in Sect. 4.1, the launch of special-rate annuities might negatively impact on the sale of standard annuities (the so called cannibalization effect). To analyze this aspect in terms of portfolio risk profile, we assume that one half of the enhanced annuity sales (subportfolio $\mathrm{SP2}$) are "subtracted" from the standard annuity business (subportfolio $\mathrm{SP1}$). Then, we consider portfolios with the following subportfolio sizes:

$$n_1 = 10\,000 - \frac{n_2}{2}$$
$$n_2 = 500, 600, \ldots, 1\,000$$
$$n_3 = 250, 300, \ldots, 500$$

Furthermore, it is reasonable to assume that, in case of a cannibalization effect, the mortality in the standard annuity subportfolio improves. To represent this aspect, we assume $M_1 = 91$ (instead of $M_1 = 90$). Results are shown in Table 5.

Table 5: Cases 1.4 - impact of the portfolio structure on the risk index

Portfolio	n_1	n_2	n_3	$\rho(n_1, n_2, n_3)$
P01	9 750	500	250	0.002340041
P02	9 700	600	300	0.002352541
P03	9 650	700	350	0.002364783
P04	9 600	800	400	0.002376774
P05	9 550	900	450	0.002388521
P06	9 500	1 000	500	0.002400030

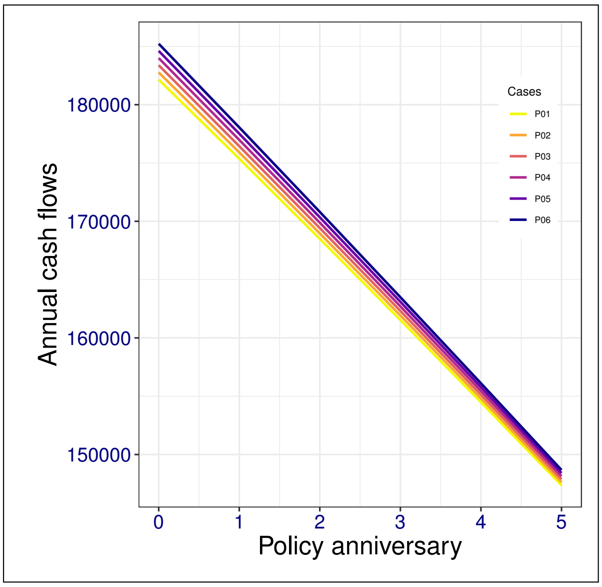


Figure 20: Cases 1.4 - impact of the portfolio structure on the annual cash flows

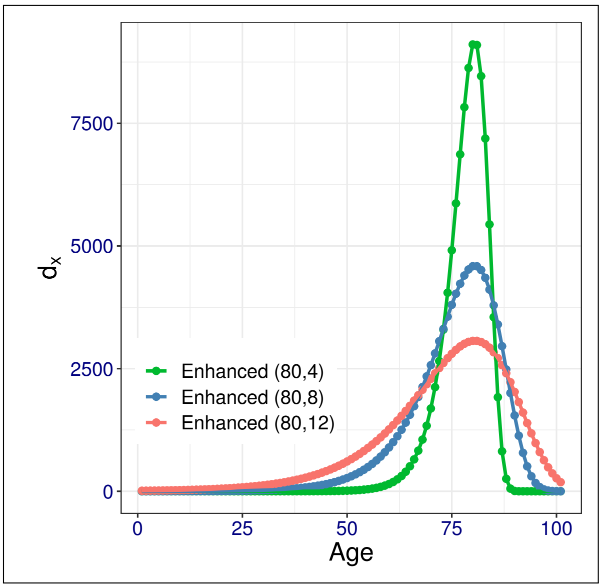
Some comments on the numerical results follow. In each set of cases, the diversity among the time profiles of the expected cash flows is self-explanatory: larger portfolio sizes imply higher amounts of benefit outflows. Conversely, the analysis of the risk index values in the various portfolio structures provides us with interesting information. We note that, in all the sets of cases we have considered, the range of values assumed by the risk index is very narrow. From a mathematical perspective, this is the straight consequence of a higher variability in terms of standard deviation (the numerator of fraction (20)) offset by a higher expected value (the denominator), and, in practice, a higher volume of premiums. A wider range of values (anyway very limited) can be noted as the effect of the number of impaired annuities: see, for example, the set of cases 1.2 where the increase in the risk index is equal to 2.6%, compared to the set 1.1 where the increase is smaller than 1%. Further interesting results, regarding the variability of the annual payouts can be achieved via stochastic analysis and are presented in Sect. 8.1.

7.3 Impact of lifetime distributions

Given the uncertainty in the biometric assumptions, a sensitivity analysis is appropriate. While keeping unchanged the parameters M_k (representing the modal age at death), we propose diverse assumptions regarding the dispersion of the lifetime distributions, which might more heavily impact on the portfolio risk profile. Hence, various assumptions on the parameters D_k are considered.

Figure 21 shows the graphs of the lifetime distribution for enhanced annuities, corresponding to different values of dispersion (parameter D) while keeping the same modal value (parameter M).

Figure 21: Three different assumptions on lifetime dispersion for enhanced annuities



Cases 2.1

We consider a portfolio only consisting of standard annuities and enhanced annuities, with given subportfolio sizes. Hence:

$$n_1 = 10\,000$$
$$n_2 = 1\,000$$
$$n_3 = 0$$

We analyze the impact of diverse assumptions on the dispersion of lifetimes in portfolio $\mathrm{SP2}$. Then:

$$D_2 = 4, 5, \ldots, 13$$

(while keeping $D_1 = 5$). Results are shown in Table 6.

Table 6: Cases 2.1 - impact of the lifetime distributions on the risk index

Portfolio	D_2	$\rho(10\,000, 1\,000, 0)$
P01	4	0.002315649
P02	5	0.002341998
P03	6	0.002364379
P04	7	0.002383918
P05	8	0.002401517
P06	9	0.002417793
P07	10	0.002433146
P08	11	0.002447834
P09	12	0.002462022
P10	13	0.002475816

Cases 2.2

We consider a portfolio only consisting of standard annuities and impaired annuities, with given subportfolio sizes.. Hence:

$$n_1 = 10\,000$$
$$n_2 = 0$$
$$n_3 = 1\,000$$

We analyze the impact of diverse assumptions on the dispersion of lifetimes in portfolio $\mathrm{SP3}$. Then:

$$D_3 = 11, 12, \ldots, 15$$

(while keeping $D_2 = 8$). Results are shown in Table 7.

Table 7: Cases 2.2 - impact of the lifetime distributions on the risk index

Portfolio	D_3	$\rho(10\,000, 0, 1\,000)$
P01	11	0.002422885
P02	12	0.002433728
P03	13	0.002444908
P04	14	0.002456358
P04	15	0.002468019

Cases 2.3

We consider a portfolio consisting of standard annuities, enhanced annuities and impaired annuities, with given subportfolio sizes. Hence:

$$n_1 = 10\,000$$
$$n_2 = 1\,000$$
$$n_3 = 500$$

We analyze the joint impact of diverse assumptions on the dispersion of lifetimes in both portfolios $\mathrm{SP2}$ and $\mathrm{SP3}$. To this purpose, we assume:

$$D_2 = D_3 = 4, 5, \ldots, 13$$

We note that lower dispersions can be achieved by restricting the range of pathologies which entitle to the purchase of enhanced annuities and impaired annuities. Results are shown in Table 8.

Table 8: Cases 2.3 - impact of the lifetime distributions on the risk index

Portfolio	$D_2 = D_3$	$\rho(10\,000, 1\,000, 500)$
P01	4	0.002360437
P02	5	0.002366549
P03	6	0.002373800
P04	7	0.002382362
P05	8	0.002392205
P06	9	0.002403223
P07	10	0.002415280
P08	11	0.002428234
P09	12	0.002441949
P10	13	0.002456293

Although dispersion in lifetime distributions may affect the risk profile of the annuity portfolio, the sensitivity analysis we have performed witness a rather limited impact on the risk index. We note that, of course, the broadest range of risk index values can be found when a portfolio consisting of standard annuities, enhanced annuities and impaired annuities is addressed, and for both the types of special-rate annuities a wide range of values for the dispersion parameter is considered.

8 Assessments of portfolios' risk profile: Stochastic approach

Deterministic assessments performed in Sect. 7 only provide values of specific markers, notably the risk index. To obtain better insights into the risk profile of a portfolio, stochastic assessments are required. To this purpose, stochastic (MonteCarlo) simulation procedures are commonly adopted. As we focus on the biometric features of the various portfolios, simulation of the number of survivors (or simulation of the individual lifetimes) is only needed.

The following Sects. 8.1 and 8.2 are organized similarly to Sects. 7.2 and 7.3, respectively. Hence, the same Cases are analyzed; nevertheless, to ease the interpretation of the results in graphical terms, we have reduced the number of alternatives.

The quantities referred to are the annual payouts from the portfolio and the portfolio fund (see Sect. 6.4). For both the quantities, empirical distributions at various times are constructed via stochastic simulation.

8.1 Impact of the portfolio structure

As already noted, we follow the organization in Cases adopted in Sect. 7.1, although reducing the number of alternatives.

Cases 1.1

We analyze the impact of the size of the subportfolio $\mathrm{SP2}$ of enhanced annuities. Then:

$$n_1 = 10\,000$$
$$n_2 = 100, 500, 1\,000$$
$$n_3 = 0$$

Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 22, 23, 24, 25, where the three portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ and $\mathrm{P03}$.

Figure 22: Cases 1.1 - Empirical distributions at time 5 of the portfolio payout

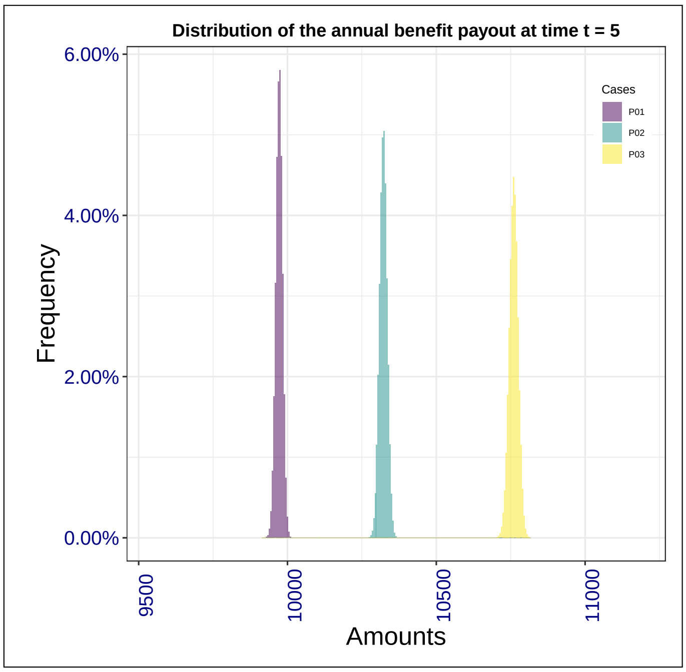


Figure 23: Cases 1.1 - Empirical distributions at time 10 of the portfolio payout

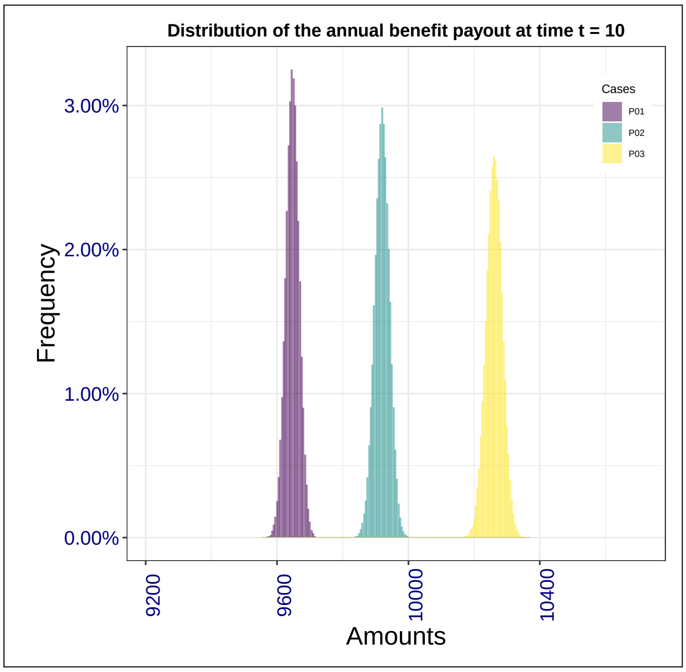


Figure 24: Cases 1.1 - Empirical distributions at time 5 of the portfolio fund

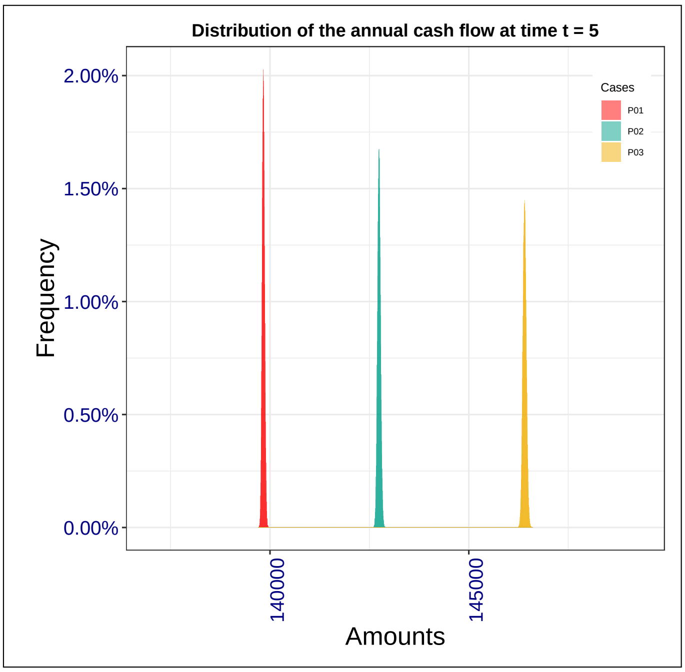
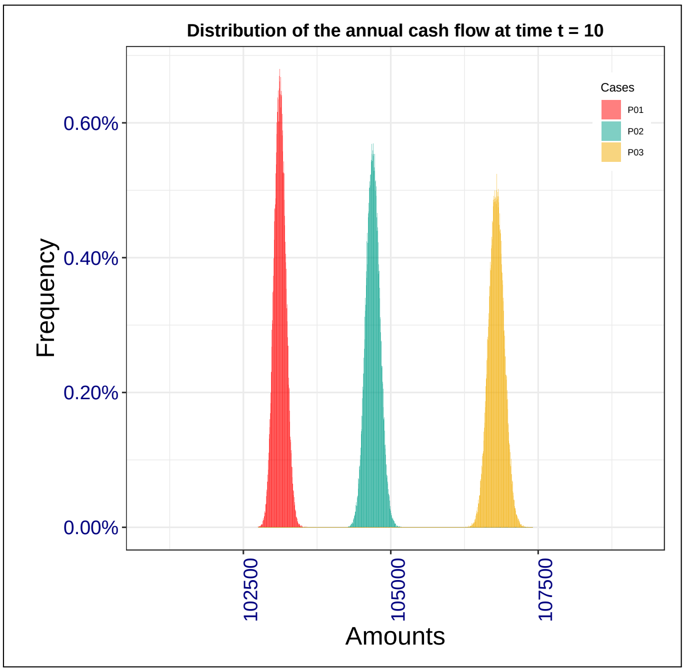


Figure 25: Cases 1.1 - Empirical distributions at time 10 of the portfolio fund



Cases 1.2

We analyze the impact of the size of the subportfolio P3 of impaired annuities. Then:

$$n_1 = 10\,000$$

$$n_2 = 0$$

$$n_3 = 100, 500, 1\,000$$

Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 26, 27, 28, 29, where the three portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ and $\mathrm{P03}$.

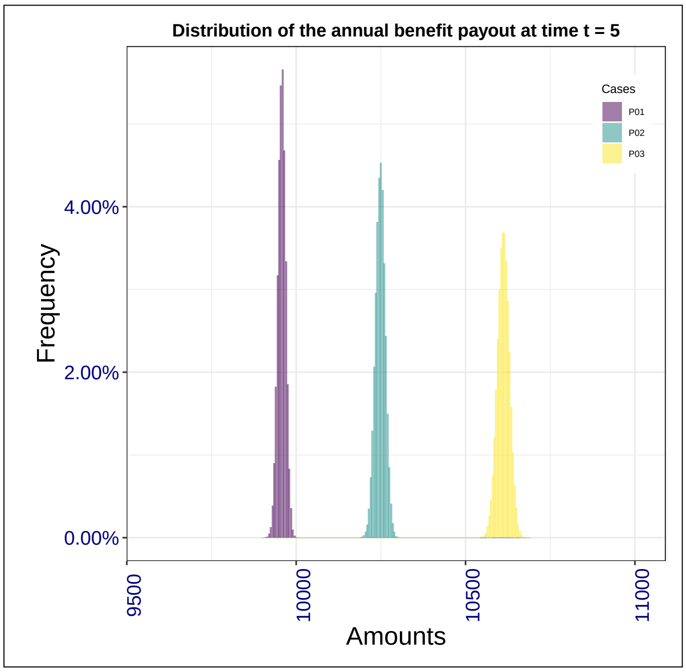


Figure 26: Cases 1.2 - Empirical distributions at time 5 of the portfolio payout

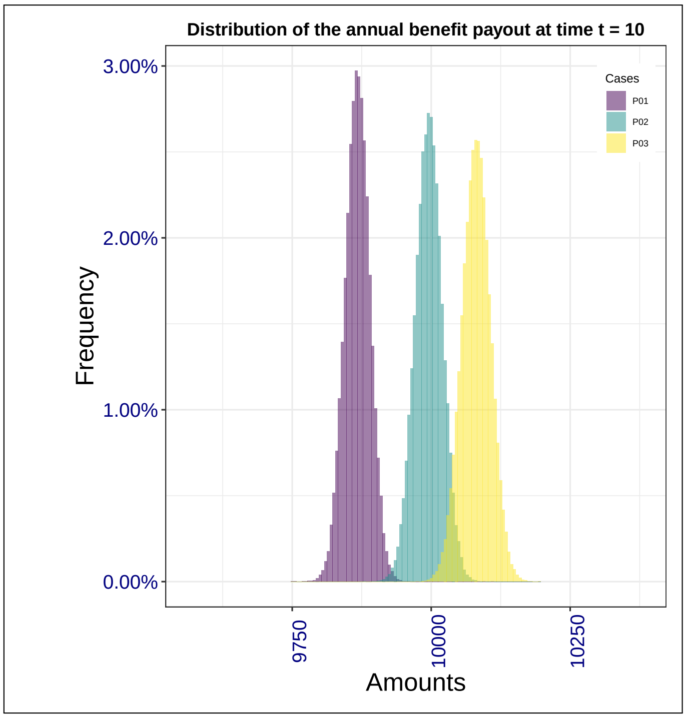


Figure 27: Cases 1.2 - Empirical distributions at time 10 of the portfolio payout

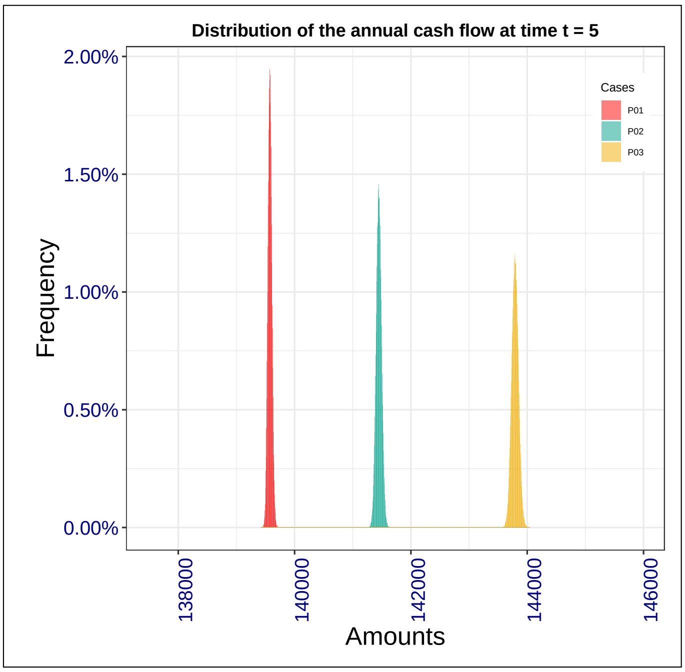


Figure 28: Cases 1.2 - Empirical distributions at time 5 of the portfolio fund

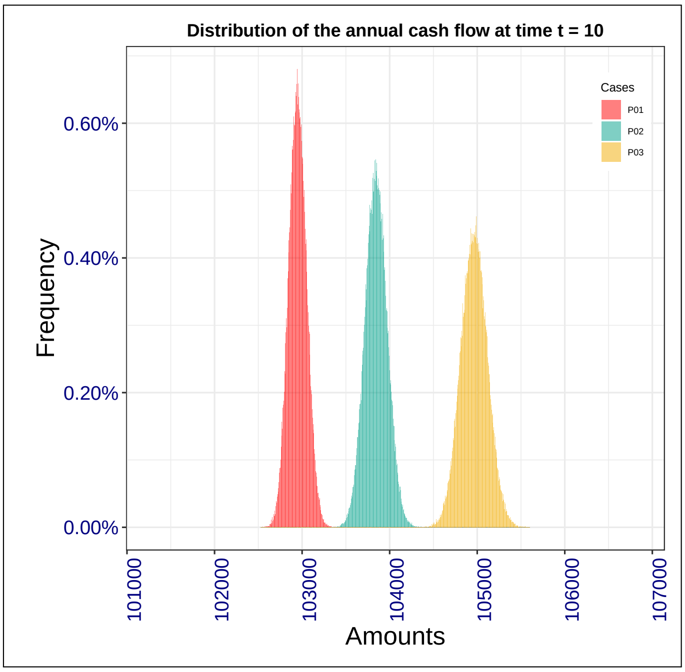


Figure 29: Cases 1.2 - Empirical distributions at time 10 of the portfolio fund

Cases 1.3

We assume that both enhanced annuities and impaired annuities are launched (together with standard annuities), and analyze the joint impact by assuming that $n_3 = n_2/2$.

Then:

$$n_1 = 10\,000$$
$$n_2 = 500, 800, 1\,000$$
$$n_3 = 250, 400, 500$$

Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 30, 31, 32, 33, where the three portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ and $\mathrm{P03}$.

Figure 30: Cases 1.3 - Empirical distributions at time 5 of the portfolio payout

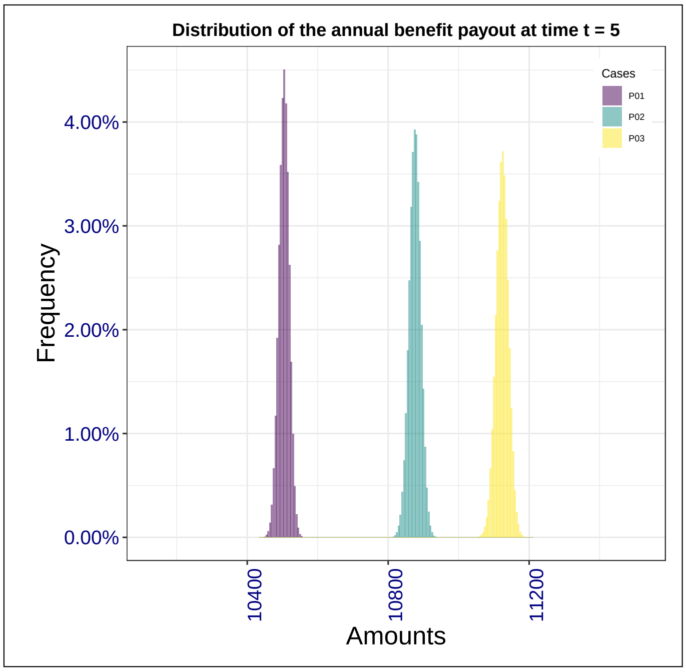


Figure 31: Cases 1.3 - Empirical distributions at time 10 of the portfolio payout

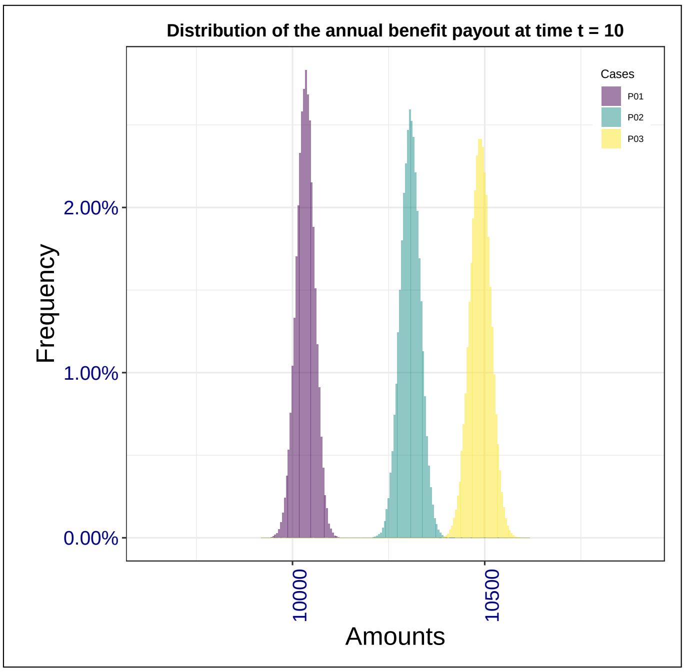
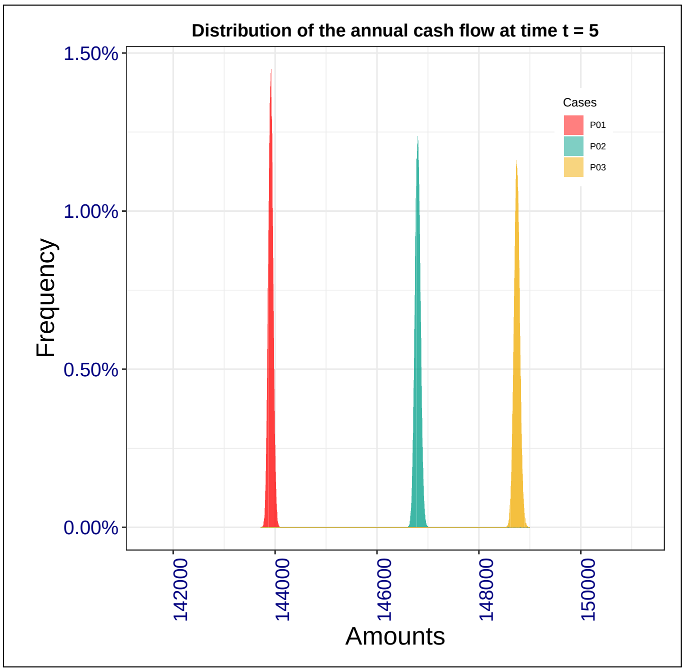


Figure 32: Cases 1.3 - Empirical distributions at time 5 of the portfolio fund



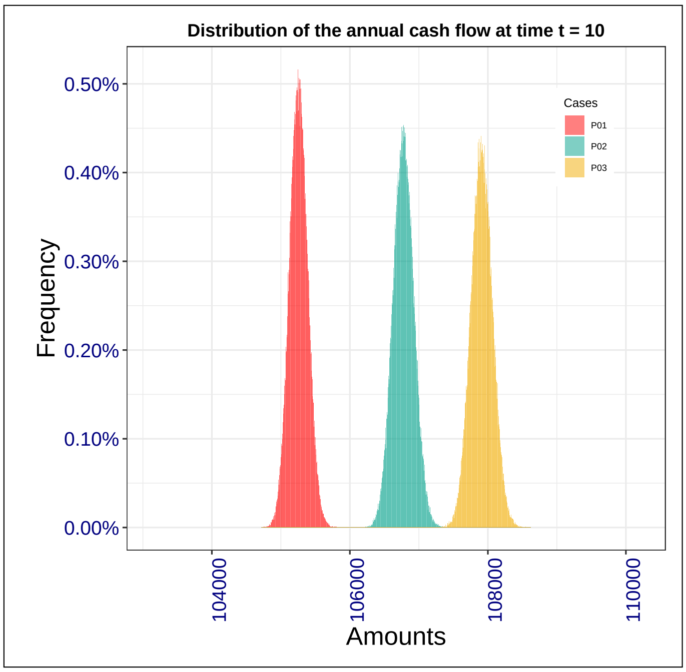


Figure 33: Cases 1.3 - Empirical distributions at time 10 of the portfolio fund

Cases 1.4

To assess the impact of a possible cannibalization effect, we consider portfolios with the following subportfolio sizes:

$$n_1 = 10\,000 - \frac{n_2}{2}$$
$$n_2 = 500, 800, 1\,000$$
$$n_3 = 250, 400, 500$$

As previously noted, to represent an improvement in mortality in the standard annuity subportfolio, we assume $M_1 = 91$ (instead of $M_1 = 90$). Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 34, 35, 36, 37, where the three portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ and $\mathrm{P03}$.

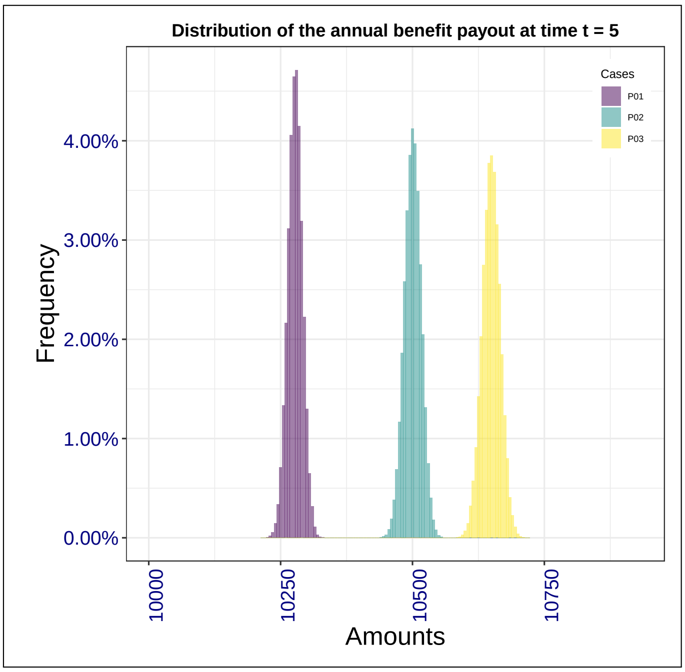


Figure 34: Cases 1.4 - Empirical distributions at time 5 of the portfolio payout

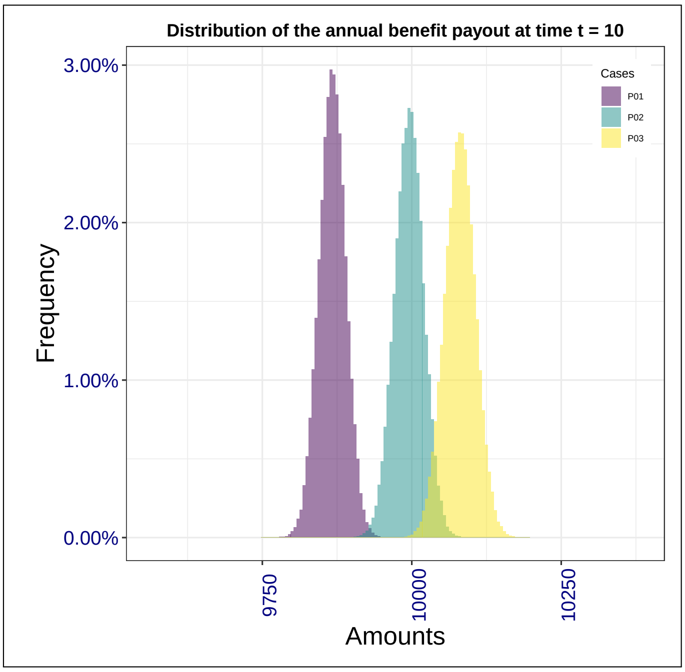


Figure 35: Cases 1.4 - Empirical distributions at time 10 of the portfolio payout

Figure 36: Cases 1.4 - Empirical distributions at time 5 of the portfolio fund

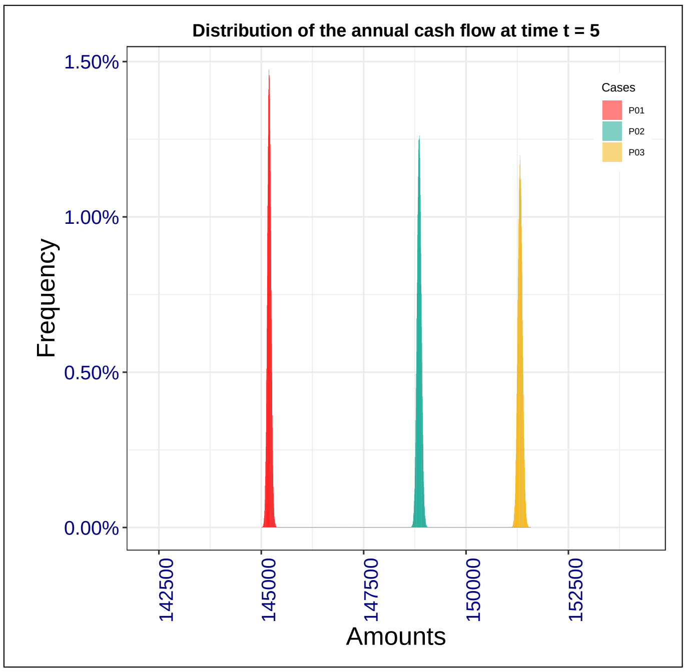
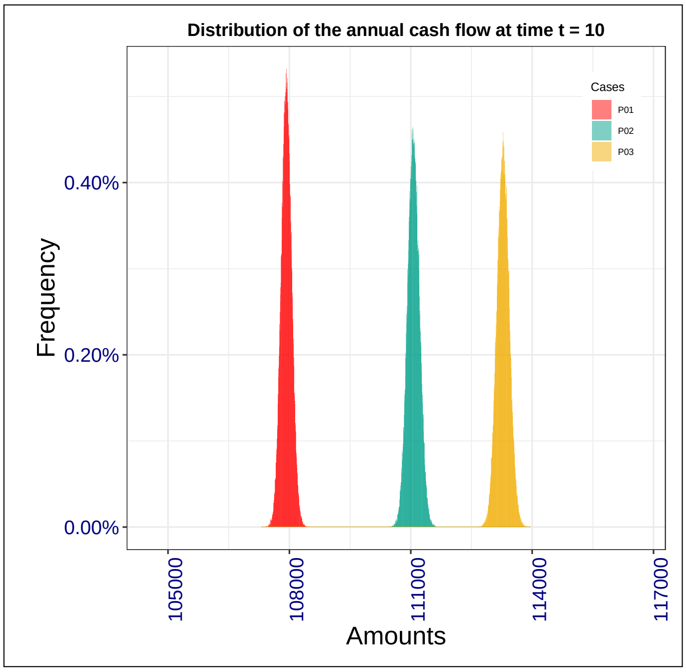


Figure 37: Cases 1.4 - Empirical distributions at time 10 of the portfolio fund



The portfolio total size and, notably, the portfolio structure (in terms of shares of standard annuities, enhanced annuities and impaired annuities) impact on the risk profile, in terms of both annual cashflows and amount of the portfolio fund.

8.2 Impact of lifetime distributions

To assess the impact of uncertainty in biometric assumptions, we analyze the Cases considered in Sect. 7.3, again with some reduction in the number of alternatives.

Cases 2.1

We consider a portfolio only consisting of standard annuities and enhanced annuities, with given subportfolio sizes. Hence:

$$n_1 = 10\,000$$
$$n_2 = 1\,000$$
$$n_3 = 0$$

We analyze the impact of diverse assumptions on the dispersion of lifetimes. Then:

$$D_2 = 4,\ 6,\ 8,\ 10,\ 12$$

(while keeping $D_1 = 5$). Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 38, 39, 40, 41, where the portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}\ldots$.

Figure 38: Cases 2.1 - Empirical distributions at times 5 of the portfolio payout

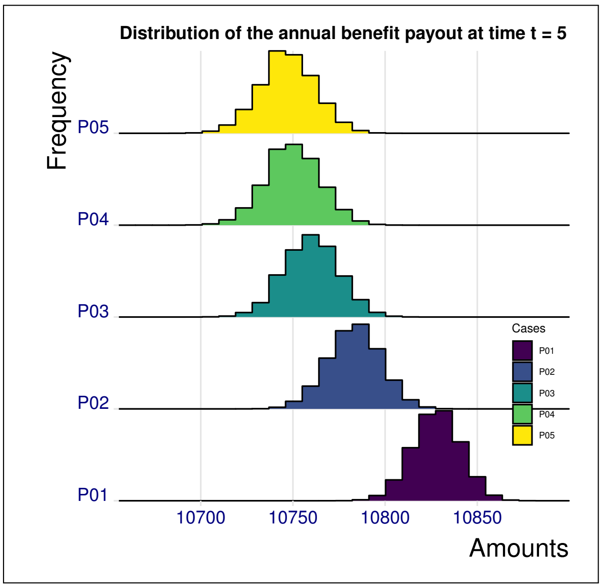


Figure 39: Cases 2.1 - Empirical distributions at times 10 of the portfolio payout

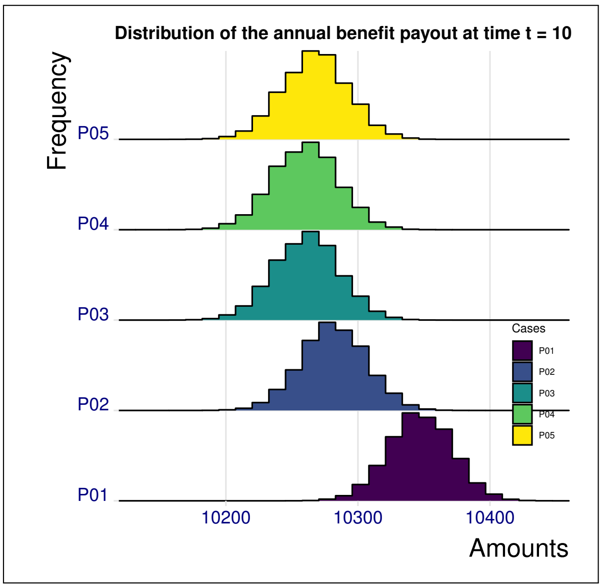
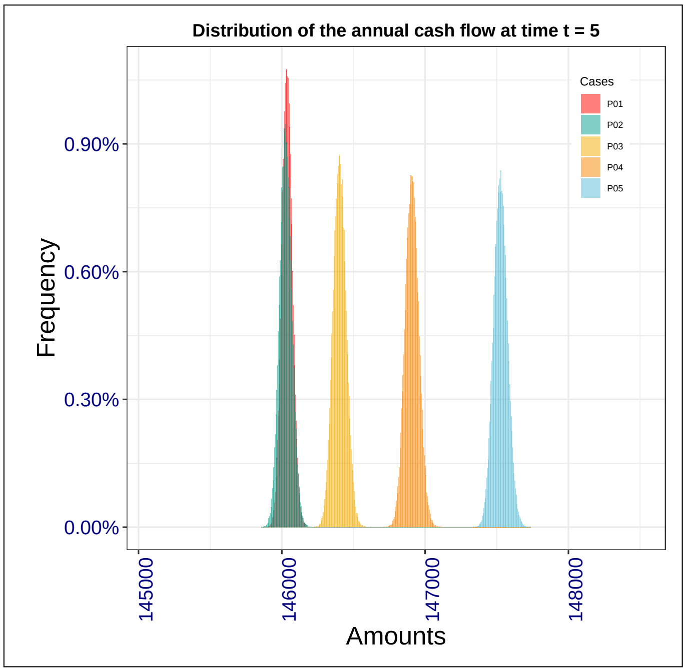


Figure 40: Cases 2.1 - Empirical distributions at time 5 of the portfolio fund



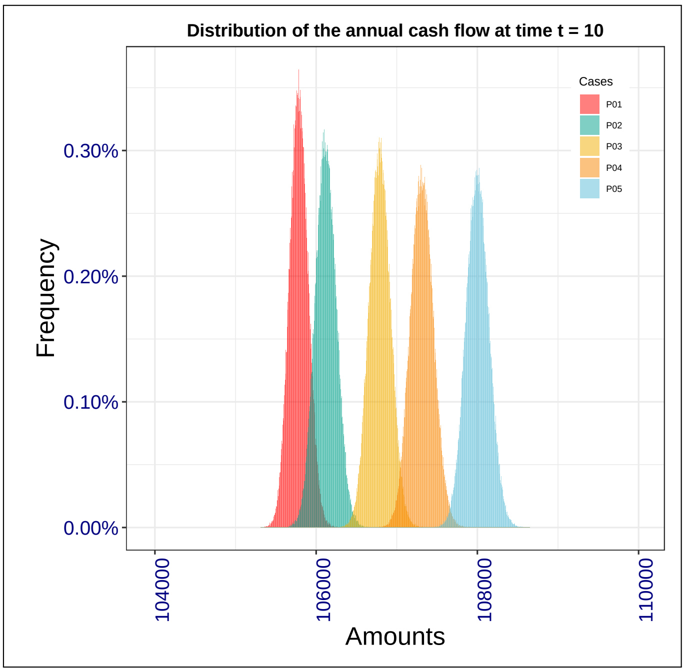


Figure 41: Cases 2.1 - Empirical distributions at time 10 of the portfolio fund

Cases 2.2

We consider a portfolio only consisting of standard annuities and impaired annuities, with given subportfolio sizes.. Hence:

$$n_1 = 10\,000$$
$$n_2 = 0$$
$$n_3 = 1\,000$$

We analyze the impact of diverse assumptions on the dispersion of lifetimes in portfolio $\mathrm{P3}$. Then:

$$D_3 = 11,\ 12, \ldots, 15$$

(while keeping $D_2 = 8$). Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 42, 43, 44, 45, where the portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ . . . .

Figure 42: Cases 2.2 - Empirical distributions at times 5 of the portfolio payout

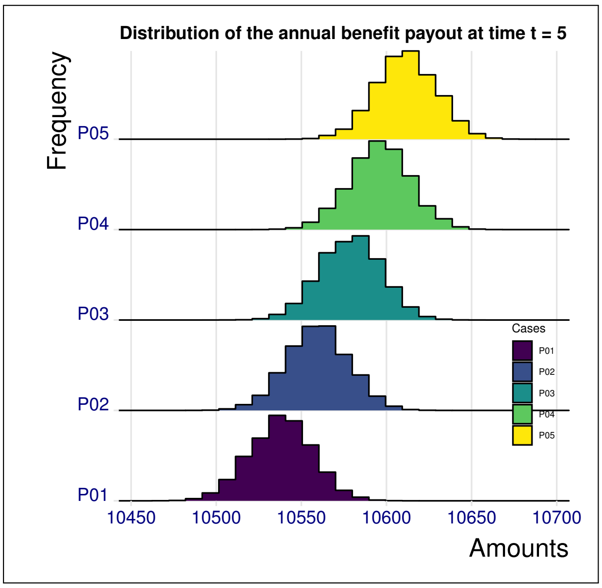


Figure 43: Cases 2.2 - Empirical distributions at times 10 of the portfolio payout

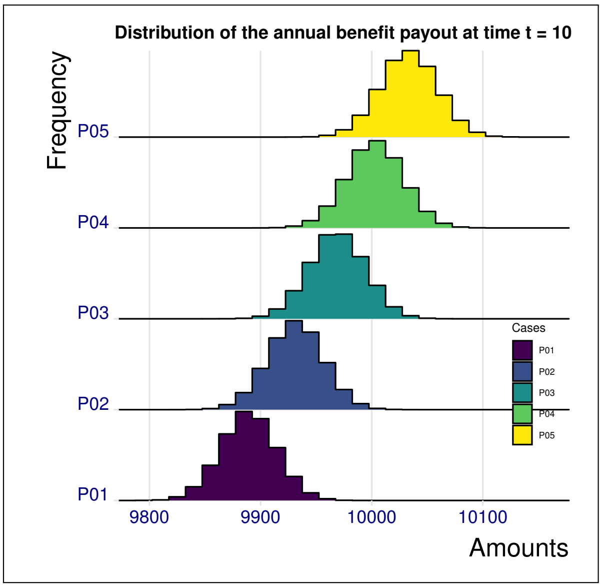


Figure 44: Cases 2.2 - Empirical distributions at time 5 of the portfolio fund

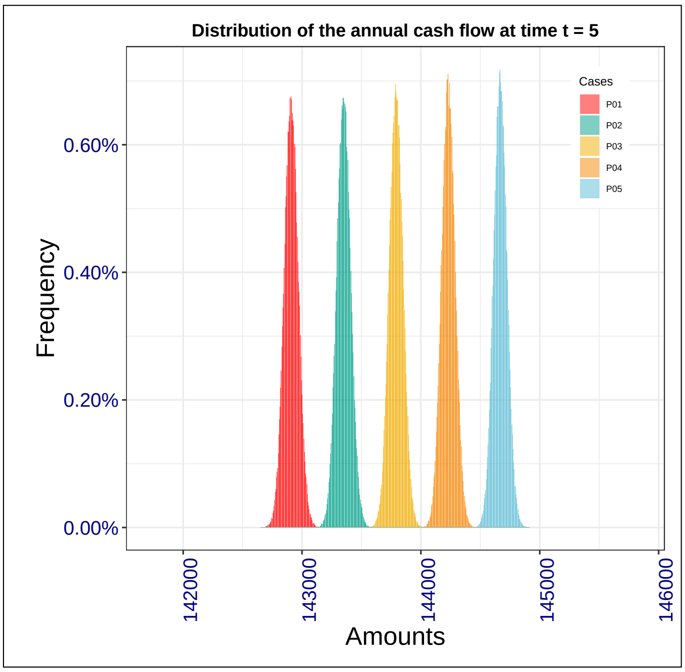
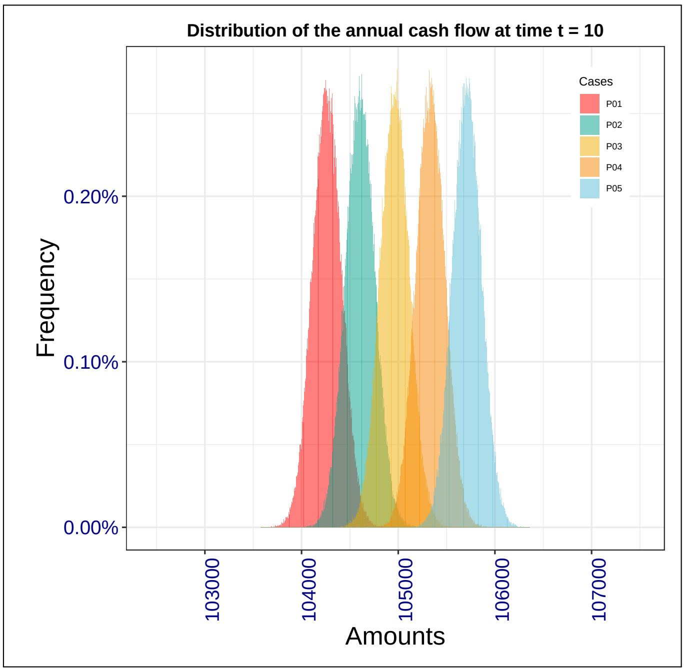


Figure 45: Cases 2.2 - Empirical distributions at time 10 of the portfolio fund



Cases 2.3

We consider a portfolio consisting of standard annuities, enhanced annuities and impaired annuities, with given subportfolio sizes. Hence:

$$n_1 = 10\,000$$
$$n_2 = 1\,000$$
$$n_3 = 500$$

We analyze the joint impact of diverse assumptions on the dispersion of lifetimes in both portfolios $\mathrm{P2}$ and $\mathrm{P3}$. To this purpose, we assume:

$$D_2 = D_3 = 8,\ 11,\ 13$$

Empirical distributions at times 5 and 10 of the portfolio payout and the portfolio fund are sketched in Figs. 46, 47, 48, 49, where the portolios are respectively denoted by $\mathrm{P01}$, $\mathrm{P02}$ . . . ..

Figure 46: Cases 2.3 - Empirical distributions at times 5 of the portfolio payout

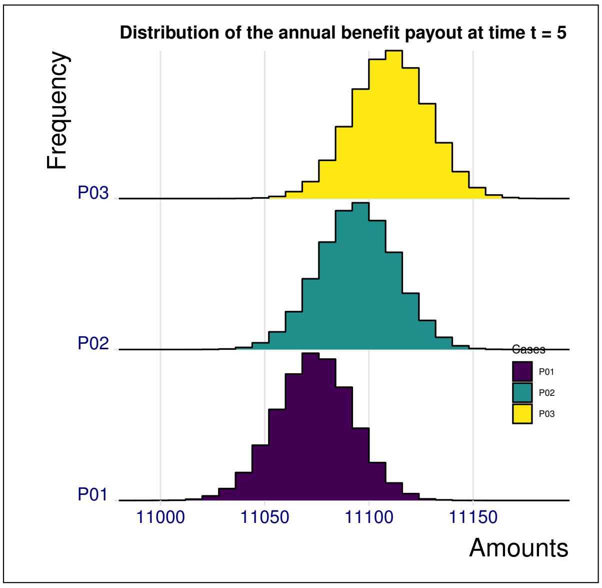


Figure 47: Cases 2.3 - Empirical distributions at times 10 of the portfolio payout

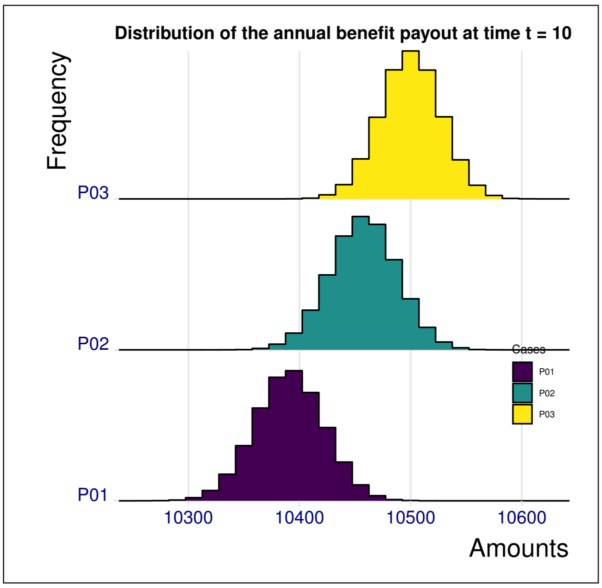


Figure 48: Cases 2.3 - Empirical distributions at time 5 of the portfolio fund

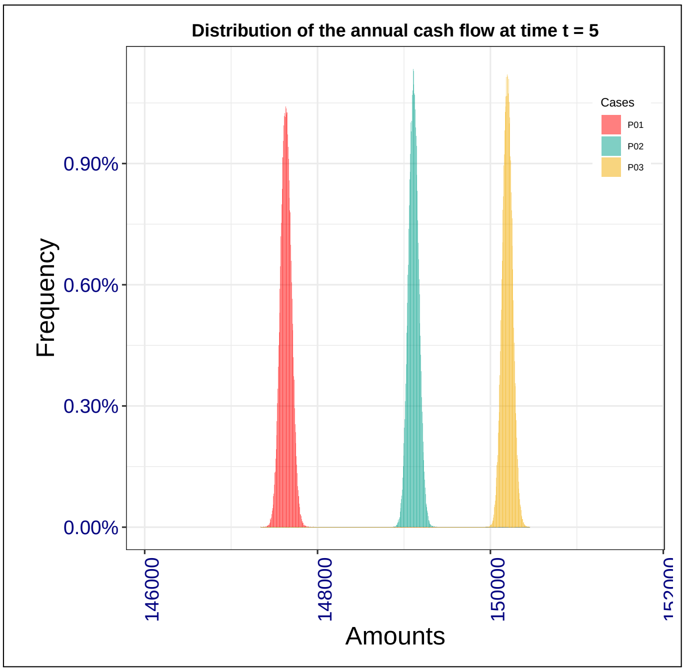
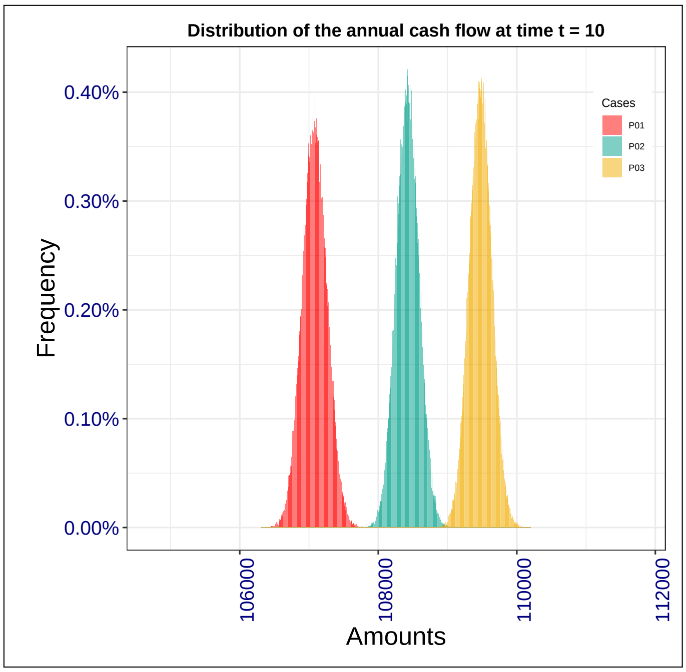


Figure 49: Cases 2.3 - Empirical distributions at time 10 of the portfolio fund



As regards the impact of lifetime distributions on monetary results of interest, that is, annual outflows and portfolio fund, achievements obtained in the stochastic setting are in line with findings from the deterministic analysis. In particular, higher variances in the lifetime distributions of course imply more dispersed distributions of results, but, as clearly appears from the various plots, increases are very limited.

9 Facing the annual payouts

Appropriate resources must be assigned to the portfolio in order to meet the annual payouts with a high probability. Diverse criteria can be adopted to quantify the above resources which, whatever the criterion adopted, will partly be provided (via the portfolio reserve) by single premiums cashed at policies issue and partly by shareholders' capital allocated to the portfolio. In what follows, we focus on the annual total amount of resources needed, disregarding the funding source.

9.1 The percentile principle

Refer to a generic portfolio. Let $X_1(t)$, $X_2(t)$, $X_3(t)$ denote the random payouts at time t, related to standard annuities, enhanced annuities and impaired annuities respectively. Let

$$X(t) = X_1(t) + X_2(t) + X_3(t) \tag{28}$$

denote the portfolio total payout at time t. We adopt the percentile principle. Hence, we have to find, for $t = 1, 2, \dots$, the amount $A(t)$ such that:

$$Pr[X(t) > A(t)] = \epsilon \tag{29}$$

where ϵ denotes an assigned (small) probability. A more detailed analysis could be performed by separately addressing the risk profile of each subportfolio, thus calculating, for $h = 1, 2, 3$ and $t = 1, 2, \dots$ the quantities $A_h(t)$ such that:

$$Pr[X_h(t) > A_h(t)] = \epsilon_h \tag{30}$$

However, we only focus on the overall requirement 29, which clearly takes into account the pooling effect.

9.2 Numerical results

We consider four portfolios with the following structures:

Table 9: Portfolio structures

Portfolio	n_1	n_2	n_3
P01	10 000	0	0
P02	10 000	1 000	0
P03	10 000	0	500
P04	10 000	1 000	500

This way, we can analyze the risk profile of a "traditional" portfolio only consisting of standard annuities (P01), a portfolio including standard annuities and enhanced annuities (P02) or standard annuities and impaired annuities (P03), and finally a portfolio including both types of special rate annuities (P04). For a given portfolio Ph, we denote with $A_{Ph}(t)$, $h = 1, 2, 3$, the amount required by the portfolio at time t. Biometric assumptions for the parameters M_k and D_k respectively for standard life annuities ($k = 1$), enhanced annuities ($k = 2$), and impaired annuities ($k = 3$) are as follows:

Table 10: Biometric assumptions:

k	M_k	D_k
1	90	5
2	80	8
3	70	13

Results in terms of assets requirements are also encouraging. We note that the range of values, expressed by the ratio between assets required and expected values of total payout, corresponding to the various portfolio structures are very limited, whatever the solvency level chosen. A higher sensibility w.r.t. the solvency level can be observed in particular for $t = 10$, because of a dispersion of the payouts increasing with time.

Figure 50: Assets backing the liabilities / Expected value at time 1

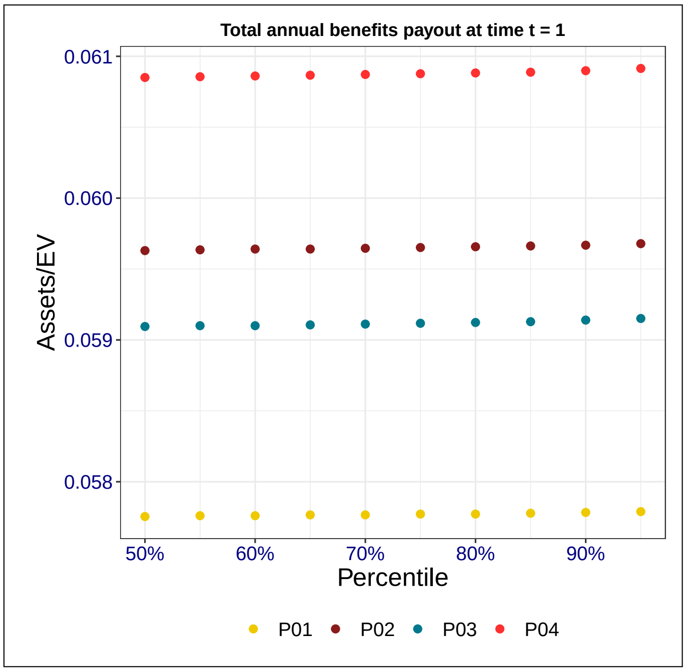


Figure 51: Assets backing the liabilities / Expected value at time 5

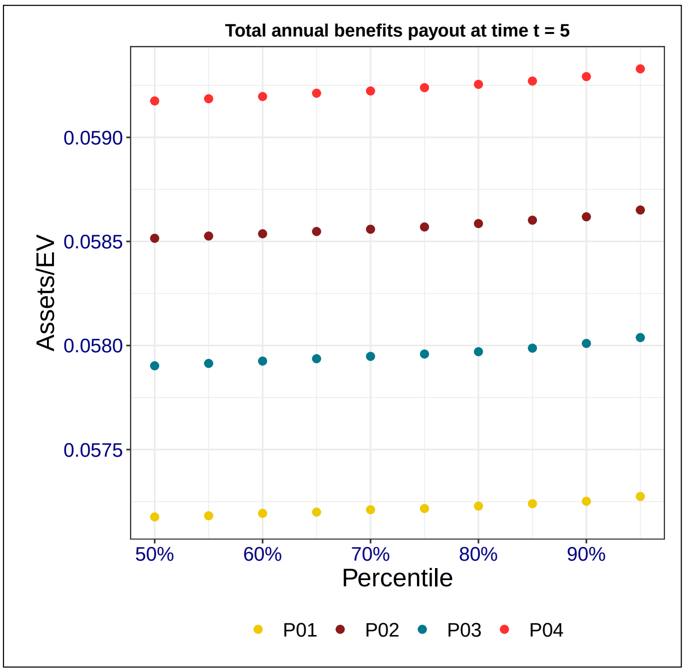
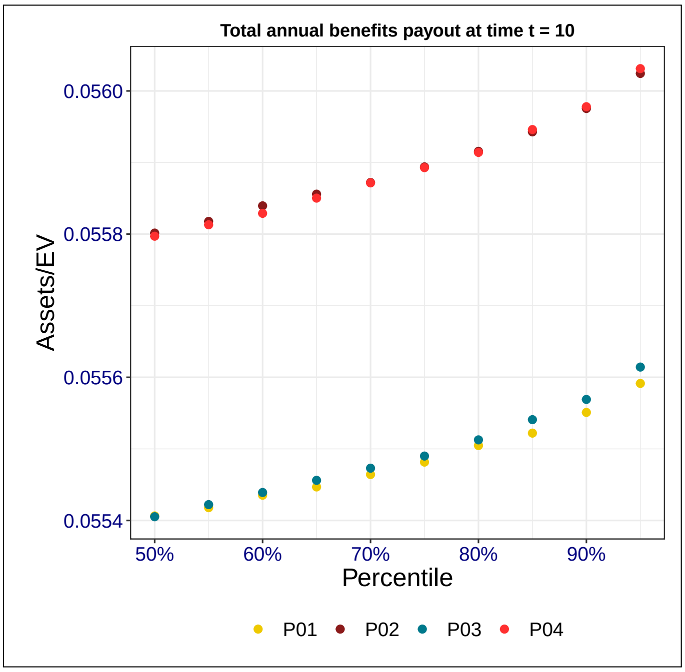


Figure 52: Assets backing the liabilities / Expected value at time 10



10 Concluding remarks

As standard life annuities are mainly attractive for healthy people, premium rates are kept high, so that a large portion of potential annuitants are out of reach of insurers. To expand their business, some insurers have started offering better annuity rates to people whose health conditions are worse than those of typical buyers of standard life annuities. Special-rate life annuity (or underwritten life annuity) products have then been designed and launched. Various rating classes are commonly defined, reflecting the health status and the consequent lifetime distribution. In our research we have focussed on enhanced life annuities and impaired life annuities, the latter referring to health conditions worse than the former.

Premium rates for underwritten life annuities must be determined and charged, according to the (assumed) lifetime probability distribution of individuals assigned to the various rating classes as a result of the underwriting step. The worse the health conditions, the smaller the modal age at death (as well as the expected lifetime), but, at the same time, the higher the variance of the lifetime distribution. The latter aspect is due to a significant data scarcity as well as to the mix of possible pathologies leading to each specific rating class. A higher degree of (partially unobservable) heterogeneity follows, inside each subportfolio of special-rate annuities. The variance of each life annuity payout of course impacts on the overall risk profile of the life annuity portfolio. Hence, on the one hand a higher premium income can be expected, on the other a higher variability of the total portfolio payout will follow because of both the larger size and the specific higher variability of payouts related to special-rate annuities.

The analysis, in quantitative terms, of the "balance" between the two aspects (that is, higher risk and higher premium income) has been the aim of this research. A number of numerical evaluations have been performed by adopting both a deterministic approach and a stochastic one as well. Diverse hypotheses on lifetime distributions have been assumed, and various portfolio sizes and structures (in terms of shares of standard annuities, enhanced annuities and impaired annuities) have been considered. Results we have obtained of course depend on assumptions (notably, regarding the portfolio structure). Nevertheless, the broad range of assumptions has allowed us to perform an effective sensitivity analysis, whose interesting achievements witness the possibility of extending the life annuity business without taking huge amounts of risk. Hence, the creation of value for customers (and hence a possible increase of the insurer's market share) can be pursued without a significant worsening of the company risk profile.

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Printed in October, 2020

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